

Moreover, we cannot have $\mu_i = -1$ because that would mean by (3) that

$$0 = \bar{z}'_i A_1 z_i + \bar{z}'_i A_2 z_i = \bar{z}'_i A z_i.$$

Relation (12) thus implies

$$(14) \quad 1 - |\mu_i|^2 > 0$$

i.e. $|\mu_i| < 1$ as was to be proved.

The part of the theorem giving the sufficient condition was already obtained by L. Seidel [1] and G. Temple in a somewhat more indirect fashion.

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SOME RECURRENCE FORMULAE IN THE INCOMPLETE BETA FUNCTION RATIO

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1. Introduction. It is well known that the incomplete beta function ratio, defined by

$$(1) \quad I_x(p, q) = \frac{B_x(p, q)}{B(p, q)},$$

where

$$(2) \quad B_x(p, q) = \int_0^x x^{p-1}(1-x)^{q-1} dx,$$

and

$$(3) \quad B(p, q) = B_1(p, q),$$