be zero for every $\alpha > 0$ and $\beta < 0$, and thus we infer from Theorem 2 that the fundamental identity holds for all real t (if the limits a_N and b_N are chosen in accordance with the conditions of this theorem). This proposition is somewhat more general than that proved in [3] by a similar method.

It also follows from the last remark and Theorem 3 that, when $P(z = 0) < 1$, (9) can be differentiated any number of times for any real t. This proposition contains the results in [2] and [3] as special cases.

7. A generalization. We finally remark that the assumption made in Theorem 3 that the expressions containing derivatives of $\varphi_\nu(t)$ are uniformly bounded is unnecessarily restrictive. For example, it seems possible to prove that the first derivative of (2) may be obtained by differentiation under the expectation sign if the series (cf. Corollary 1 to Theorem 7.4. in [6])

$$\sum_{m=1}^{\infty} P(n = m) \sum_{\nu=1}^{m} \frac{\varphi_\nu'(t)}{\varphi_\nu(t)}$$

is uniformly convergent with respect to t.

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SPREAD OF MINIMA OF LARGE SAMPLES

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1. Theorems. Let x have the continuous cumulative distribution function $F(x)$. Let $(x_1, \cdots, x_N)$ be a sample of N independent values of x and $y = \inf(x_1, \cdots, x_N)$. Then y is a random variable with the cumulative distribution function

$$G_N(y) = 1 - (1 - F(y))^N. \tag{1}$$

Let K values of the new variable y be drawn, $(y_1, \cdots, y_K)$ and let the spread

$$w = \sup(y_1, \cdots, y_K) - \inf(y_1, \cdots, y_K).$$