A NOTE ON FISHER'S INEQUALITY FOR BALANCED INCOMPLETE BLOCK DESIGNS

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1. An experimental design in which v varieties or treatments are arranged in b blocks, is called a *balanced incomplete block design* if

(i) Each block has exactly k treatments ($k < v$) no treatment occurring twice in the same block.

(ii) Each treatment occurs in exactly r blocks.

(iii) Any two treatments occur together in exactly λ blocks.

It is easy to see that the parameters v, b, r, k, λ of the design satisfy the relations

$$bk = vr \tag{1.0}$$

$$\lambda(v - 1) = r(k - 1). \tag{1.1}$$

Also it is readily seen that

$$r > \lambda \tag{1.2}$$

for otherwise with any given treatment every other treatment would occur in every block. This would make $k = v$, and the design would become a 'randomised block design'.

Fisher (1940), showed that a necessary condition for the existence of a balanced incomplete block design with v treatments and b blocks is

$$b \geqq v. \tag{1.3}$$

It is the object of this note to give a very simple proof of Fisher's inequality.

2. Consider a balanced incomplete block design with parameters

$$v, b, r, k, \lambda \tag{2.0}$$

and let

$$n_{ij} = 1 \text{ or } 0 \tag{2.1}$$

according as the ith treatment does or does not occur in the jth block. Clearly

$$\sum_{j=1}^{b} n_{ij}^2 = r \tag{2.2}$$

$$\sum_{j=1}^{b} n_{ij} n_{i'j} = \lambda \qquad (i \neq i'). \tag{2.3}$$