ADJUSTMENT OF AN INVERSE MATRIX CORRESPONDING TO A CHANGE IN ONE ELEMENT OF A GIVEN MATRIX

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1. Introduction. Many methods have been published in recent years for carrying out the numerical computation of the inverse of a matrix [1], [2]. In all these methods, the amount of computation increases rapidly with increase in order of the matrix.

The utility of a computational method for obtaining the inverse of a matrix would be increased considerably if the inverse could be transformed in a simple manner, corresponding to some specified change in the original matrix, thus eliminating the necessity of computing the new inverse from the beginning. The problem that is considered in the present paper is one of changing one element in the original matrix, and of computing the resulting changes in the elements of the new inverse directly from those of the old inverse.

2. Computational method. Let

a_{ij}, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, n$ denote the elements of an nth order square matrix $\boldsymbol{a}$;

b_{ij}, denote the elements of $\boldsymbol{b}$, the inverse of $\boldsymbol{a}$;

A_{ij}, denote the elements of $\boldsymbol{A}$ which differs from $\boldsymbol{a}$ only in one element, say A_{RS};

B_{ij}, denote the elements of $\boldsymbol{B}$, the inverse $\boldsymbol{A}$.

Let

$$A_{RS} = a_{RS} + \Delta a_{RS}.$$

The set of equations by means of which $\boldsymbol{B}$ may be computed from Δa_{RS} and $\boldsymbol{b}$ is

$$B_{rj} = b_{rj} - \frac{b_{rR}\, b_{Sj}\, \Delta\, a_{RS}}{1 + b_{SR}\, \Delta\, a_{RS}}, \qquad r = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, n, \tag{1}$$

provided that $1 + b_{SR}\Delta a_{RS} \neq 0$.

The validity of equation (1) may be demonstrated by multiplying through by A_{ir}, $(r = 1, 2, \cdots, n)$ and adding the results:

$$\sum_{r=1}^{n} A_{ir}\, B_{rj} = \sum_{r=1}^{n} A_{ir}\, b_{rj} - \frac{b_{Sj}\, \Delta\, a_{RS}}{1 + b_{SR}\, \Delta\, a_{RS}} \sum_{r=1}^{n} A_{ir}\, b_{rR}, \quad (i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, n). \tag{2}$$

Consider separately the equations for which $i \neq R$, and for which $i = R$.

Case I. $i \neq R$. By hypothesis, $A_{ir} = a_{ir}$ for $i \neq R$. Hence equations (2) become

$$\sum_{r=1}^{n} A_{ir}\, B_{rj} = \sum_{r=1}^{n} a_{ir}\, b_{rj} - \frac{b_{Sj}\, \Delta\, a_{RS}}{1 + b_{SR}\, \Delta\, a_{RS}} \sum_{r=1}^{n} a_{ir}\, b_{rR}, \quad (i = 1, 2, \cdots, R-1, R+1, \cdots, n;\ j = 1, 2, \cdots, n). \tag{3}$$