For statistics having normal sampling distributions such a ratio would be independent of α and would be equivalent to the ratio of the variances of these sampling distributions. It was found that δ_α^2 is independent of α except for a maximum change of 1 in the second decimal for the values of $\alpha = .005, .01, .025, .05, .10$. These values of δ^2 are presented in Table 3 along with the relative precision of the range as an estimate of σ as given by Mosteller [1].

It is interesting to note that δ^2 corresponds very closely to the relative precision of the range as an estimate of σ.

REFERENCE

[1] F. Mosteller, "On some useful 'inefficient' statistics," *Annals of Math. Stat.*, Vol. 17 (1946), pp. 377–408.

A NOTE ON THE ESTIMATION OF A DISTRIBUTION FUNCTION BY CONFIDENCE LIMITS

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Let $F(x)$ be the continuous cumulative distribution function of a random variable X, and let $x_1 < x_2 < x_3 < \cdots < x_n$ be the results of n independent observations on X arranged in order of size. We wish to estimate $F(x)$ by means of the band $S_n(x) \pm \lambda/\sqrt{n}$ where $S_n(x)$ is defined by

$$S_n(x) = \begin{cases} 0 & \text{if } x < x_1, \\ k/n & \text{if } x_k \le x < x_{k+1}, \\ 1 & \text{if } x \ge x_n. \end{cases}$$

Thus we wish to know the probability, say $P_n(\lambda)$, that the band is such that $S_n(x) - \dfrac{\lambda}{\sqrt{n}} < F(x) < S_n(x) + \dfrac{\lambda}{\sqrt{n}}$ for all x. This problem has been previously studied [1] [2] [3] [4] [5] and a limiting distribution has been obtained [1] [4] [5] and tabled [3] [4]. However apparently no error terms for the limiting distribution, or practical methods of obtaining $P_n(\lambda)$ have been given. Such a method is given here.

It has been shown [2] that $P_n(\lambda)$ is independent of $F(x)$ provided only that $F(x)$ is continuous, and thus it is sufficient to consider only the case

$$F(x) = \begin{cases} 0 \text{ if } x < 0, \\ x \text{ if } 0 \le x \le 1, \\ 1 \text{ if } x \ge 1. \end{cases}$$

We will find the probability that $S_n(x)$ falls wholly in the band $F(x) \pm k/n$ (here $\lambda = k/\sqrt{n}$) where k is an integer or a rational number, and intermediate values may be obtained by interpolation. To illustrate the method we shall assume that k is an integer.