ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Chicago meeting of the Institute, April 28–29, 1950*)

1. The Distribution of the Quotient of Ranges in Samples from a Rectangular Population. PAUL R. RIDER, Washington University, St. Louis, Missouri.

The distribution of the quotient of the ranges of two independent, random samples from a continuous rectangular population is derived. The distribution is independent of the population range and can be used to test the hypothesis that two samples came from the same rectangular population just as the distribution of the variance ratio is used to test whether two samples came from the same normal population.

2. A Geometric Method for Finding the Distribution of Standard Deviations when the Sampled Population Is Arbitrary. (Preliminary Report). PAUL IRICK, Purdue University.

For an ordered random sample, $x_1 \leqq x_2 \leqq \cdots \leqq x_n$, chosen from a population, $f(x)$, $a \leqq x \leqq b$, let $r_i = x_{i+1} - x_i \geqq 0, i = 1, 2, \cdots, n - 1$. Make the transformation

$$r_i = -\sqrt{\frac{i-1}{2i}}\, r'_{i-1} + \sqrt{\frac{i+1}{2i}}\, r'_i,$$

and call U' the $1/n!$ portion of the r' space bounded by the $n - 1$ sphere and hyperplanes, $\sum_1^{n-1} r_i'^2 = 2ns^2$, $r'_i = \sqrt{\frac{i-1}{i+1}}\, r'_{i-1}$, $i = 1, 2, \cdots, n - 1$, where s is the sample standard deviation. The point density in U', $\delta(r')$, is the transform of

$$\delta(r) = \int_{x_1=a}^{b-\Sigma r_i} f(x_1)f(x_1 + r_1) \cdots f(x_1 + r_1 + \cdots + r_{n-1})\, dx_1 .$$

Change to generalized polar coordinates and call U the outer hyperspherical boundary of U' whereon the density is designated by $\delta(\sqrt{2n}s, \varphi)$. Then $p(s)$, the probability law for s, is given by

$$p(s)\, ds = n!\, n^{n/2} s^{n-2}\, ds \int_{\varphi_1} \cdots \int_{\varphi_{n-2}} \delta(\sqrt{2n}s, \varphi) \sin^{n-3} \varphi_1 \cdots \sin \varphi_{n-3}\, d\varphi_{n-2} \cdots d\varphi_1,$$

where

$$\text{arc cos} \sqrt{\frac{n}{(n-i)(i+1)}} \leqq \varphi_i \leqq \text{arc cos} \left[\sqrt{\frac{i-1}{i+1}} \Big/ \tan \varphi_{i-1}\right], i = 1, 2, \cdots, n - 2,$$

whenever b is infinite. The distribution of sample range is readily found in U' and is expressible in the same form as $p(s)$ with the same limits of integration. When b is finite, the complete integral holds only for $0 \leqq s \leqq (b - a)/\sqrt{2n}$, there being $n^2/4$ connected arcs in $p(s)$ if n is even, and $(n^2 - 1)/4$ arcs if n is odd. The axes are rotated to give relatively simple formulas for $p(s)$ when $n \leqq 4$, the case of $n = 5$ also being discussed. The method readily produces previously reported results for $p(s)$. In the application of the method, particular attention has been paid to the Type III and polynomial Type I populations. The density function provides much information concerning the form of $p(s)$ for various populations, and contours of constant δ in U' are of theoretical interest.