Since $\sigma^2 > u'^2$, this is the characteristic function for two variables which are normally distributed. Thus, the simultaneous distribution of ξ and M is asymptotically normal. It is of interest to note that, if the pdf $f(x)$ is symmetric, the correlation coefficient is zero, and M and ξ are asymptotically independent. We might also note that $\phi(t_1, 0)$ is the characteristic function for the mean deviation from the sample median. Thus, the random variable M is asymptotically normal with asymptotic mean and variance u' and $((m - \theta)^2 + \sigma^2 - u'^2)/2k$ respectively.

The author wishes to express his appreciation to Professor A. T. Craig for valuable suggestions in the study of this problem.

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NOTE ON THE EXTENSION OF CRAIG'S THEOREM TO NON-CENTRAL VARIATES

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A theorem due to A. T. Craig [1] and H. Hotelling [3] concerning the distribution of real quadratic forms in normal variates is extended to the case of non-central normal variates with equal variance.

The following notation is used: A, A_1, A_2 are real symmetric matrices, L is an orthogonal matrix, Γ is a diagonal matrix of latent roots, and X, Y, M and U are column vectors.

Theorem. *Let $X' = (x_1, \cdots, x_n)$ be a set of normally and independently distributed variates with equal variance σ^2 and means $M' = (m_1, \cdots, m_n)$. Then, a necessary and sufficient condition that a real symmetric quadratic form $Q(X) = X'AX$ of rank r be distributed as $\sigma^2\chi^2$, where*

$$p(\chi^2, r, \lambda^2) = \tfrac{1}{2}e^{-\lambda^2}(\chi^2/2)^{(r-2)/2}\, e^{-\chi^2/2} \tag{1}$$
$$\sum_{j=0}^{\infty} (\lambda^2\chi^2/2)^j/j!\,\Gamma[(r - 2j)/2],$$

is that $A^2 = A$. If $Q(X)/\sigma^2$ is distributed by $p(\chi^2, r, \lambda^2)$, then $\lambda^2 = Q(M)/2\sigma^2$.

Further, let $Q_1(X) = X'A_1X$ and $Q_2(X) = X'A_2X$ be real symmetric quadratic forms of ranks r_1 and r_2. Then a necessary and sufficient condition that $Q_1(X)$ and $Q_2(X)$ be statistically independent is that $A_1A_2 = 0$.

Proof. The theorem is proved by establishing the equivalence and factorization of moment generating functions [4]. The moment generating function of