It can also be proved, by considering the limiting form of the recurrence relation (19), that the frequency function f_n is asymptotically normal. The main difficulty of proving this fact lies in showing that the frequency function actually possesses a limiting form; and the proof is rather too long to be given here.

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A NOTE ON THE ASYMPTOTIC SIMULTANEOUS DISTRIBUTION OF THE SAMPLE MEDIAN AND THE MEAN DEVIATION FROM THE SAMPLE MEDIAN

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Consider a random sample of $2k + 1$ values from a one-dimensional distribution of the continuous type with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x) = F'(x)$. Let the mean, standard deviation and median of the distribution be denoted by m, σ and θ respectively (θ assumed to be unique). We shall suppose that in some neighborhood of $x = \theta$, $f(x)$ has a continuous derivative $f'(x)$.

If we arrange the sample values in ascending order of magnitude:

$$x_1 < x_2 < \cdots < x_{2k+1},$$

there is a unique sample median x_{k+1} which we shall denote by ξ. The mean deviation from the sample median is then defined by

$$M = \frac{1}{2k} \sum_{i=1}^{2k+1} | x_i - \xi |.$$

In the material that follows we shall assume that the sample items have been ordered only to the extent that k of them are less than ξ and k of them are greater than ξ.

We then have the following

Theorem. *Let $f(x)$ be a pdf with finite second moment, continuous at $x = \theta$ with $f(\theta) \neq 0$. Then the simultaneous distribution of ξ and M is asymptotically normal. The means of the limiting distribution are θ, the population median, and u', the mean deviation from the population median, while the asymptotic variances are $1/4f^2(\theta)2k$ and $((m - \theta)^2 + \sigma^2 - u'^2)/2k$. The asymptotic expression for the correlation coefficient is $(m - \theta)/\sqrt{(m - \theta)^2 + \sigma^2 - u'^2}$.*

Proof: Let $u = (M - u')\sqrt{2k}$ and $v = (\xi - \theta)\sqrt{2k}$, where $u' = E | x - \theta |$. Then the simultaneous characteristic function of the two random variables u

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