NOTES

This section is devoted to brief research and expository articles and other short items.

A NOTE ON THE POWER OF A NON-PARAMETRIC TEST

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1. Introduction. Let $x_1 < x_2 < \cdots < x_n$ be the ordered results of n independent observations of a random variable X which has a continuous cumulative distribution function $F(x)$. The following test for the hypothesis that $F(x)$ has some specified form, say $F_0(x)$, has been suggested by Wolfowitz [1].

Form the cumulative distribution of the sample and obtain the maximum deviation of this from $F_0(x)$. Thus if

$$\begin{aligned} S_n(x) &= 0 && \text{when } x < x_1, \\ &= \frac{k}{n} && \text{when } x_k \leq x < x_{k+1}, \\ &= 1 && \text{when } x_n < x, \end{aligned}$$

the test statistic used would be

$$d = \max_x | F_0(x) - S_n(x) | \sqrt{n},$$

and the hypothesis would be rejected if d is large, say larger than d_α which is so chosen that the probability of a type I error is α. The limiting distribution (as $n \to \infty$) of d has been tabled [2], and a short table of the distribution of d for various small values of n ($n \leq 80$) has been given [3].

The purpose of this note is as follows: 1. A lower bound for the power of the test is given. 2. This test is shown to be consistent against any continuous alternative $F(x) = F_1(x)$, where $F_1(x) \not\equiv F_0(x)$. 3. The test is shown to be biased for finite n. 4. An indication of similar results for a two sample test.

2. Lower bound for the power function. Let $\Delta = \max_x | F_0(x) - F_1(x) |$ and let x_0 be a value of x such that $\Delta = | F_0(x_0) - F_1(x_0) |$. The probability that $d > d_\alpha$ is certainly not less than $\Pr\{\sqrt{n} | F_0(x_0) - S_n(x_0) | > d_\alpha\}$. This is the same as

$$1 - \Pr\left\{F_0(x_0) - \frac{d_\alpha}{\sqrt{n}} < S_n(x_0) < F_0(x_0) + \frac{d_\alpha}{\sqrt{n}}\right\},$$

which, since $S_n(x_0)$ is the proportion of observations falling less or equal to x_0, is given by the binomial probability law.

If $F(x) = F_1(x)$ the probability of an observation being less than x_0 is $F_1(x_0)$. Since $F_0(x_0) = F_1(x_0) \pm \Delta$ the above probability can be written as follows: