DISTRIBUTION OF MAXIMUM AND MINIMUM FREQUENCIES IN A SAMPLE DRAWN FROM A MULTINOMIAL DISTRIBUTION

By Robert E. Greenwood and Mark O. Glasgow

University of Texas

1. Introduction. In this paper, the expected values

$$(1.1)\quad E\begin{bmatrix}\max\\ \min\end{bmatrix}(n_1, n_2, \cdots, n_l) = \sum_{n_1+n_2+\cdots n_k=N} \frac{N!}{n_1!n_2!\cdots n_k!}\begin{bmatrix}\max\\ \min\end{bmatrix}(n_1, n_2, \cdots, n_l)\cdot p_1^{n_1}p_2^{n_2}\cdots p_k^{n_k}$$

will be studied. The quantities $\{n_i\}$, $i = 1, 2, \cdots, k$, are understood to be non-negative integers, and the quantities $\{p_i\}$ are non-negative probabilities, $\Sigma p_i = 1$. Also, $l \leqq k$. Form (1.1) will be evaluated for the binomial case $l = k = 2$ and for the special trinomial case $p_1 = p_2$ with $l = 2, k = 3$.

2. Binomial distribution. The evaluations for the expected values in the binomial case can be given explicitly in terms of the incomplete Beta function. This function may be defined by the relation

$$(2.1)\quad I_q(n - k, k + 1) = \sum_{r=0}^{k} \binom{n}{r} (1 - q)^r q^{n-r},$$

whence

$$I_{1-q}(k + 1, n - k) = \sum_{r=k+1}^{n} \binom{n}{r} (1 - q)^r q^{n-r}.$$

It is seen that

$$(2.2)\quad I_q(n - k, k + 1) + I_{1-q}(k + 1, n - k) = 1.$$

For the binomial case, $n_2 = N - n_1$ and $p_2 = 1 - p_1$, and thus instead of (n_1, n_2) and (p_1, p_2) one may use $(n, N - n)$ and $(p, 1 - p)$ without any subscripts and without sacrifice of clarity. This will be done in some instances in what follows. The evaluation of

$$(2.3)\quad E\begin{bmatrix}\max\\ \min\end{bmatrix}(n_1, n_2) = \sum_{n=0}^{N} \binom{N}{n}\begin{bmatrix}\max\\ \min\end{bmatrix}(n, N - n)\, p^n(1 - p)^{N-n}$$

is slightly different for the two cases N odd and N even.

For N odd, and for the minimum form, the summation may be written in two parts, (a) and (b),

$$(a)\quad 0 \leqq n \leqq \frac{N - 1}{2},$$