

$0 \leq q \leq 1$. (3) $x_1 \leq 0, x_n \geq 0, q \leq 0$, culminating in the general case: (4) $-\omega_1 \leq x_1 \leq x_n \leq \omega_2, -\omega_2/\omega_1 \leq q < \infty$. The common procedure in the first three cases is to integrate out the extreme from the joint distribution of one extreme and the extremal quotient. Geometric considerations give the appropriate regions of integration. The general case is obtained by a composition of cases (3), (2), and (1). For symmetrical initial distributions there exist only two branches which join at $q = 1$, and the probability function may be written in a symmetrical form. When $n = 2$, the distribution of q for a symmetrical distribution is symmetrical about zero and invariant under a reciprocal transformation, and if the initial distribution possesses no moments and does not vanish at $x = 0$, the density of probability becomes infinite at $q = 0$. The distribution of q is not affected by changes in scale but is very sensitive to changes in origin. For a uniform distribution, the extremal quotient of a nonnegative variate has just the opposite qualities of the extremal quotient of a nonpositive variate. For variates changing sign, the extremal quotient is asymptotically negative.

31. The Distributions of the t and F Statistics for a Class of Nonnormal Populations. RALPH A. BRADLEY, Virginia Polytechnic Institute.

Series expansions of the cumulative distribution functions of t and of F in powers of t^{-1} and F^{-1} are obtained. The general method of derivation presented is valid for populations with density functions, $f(u)$, such that $f(u) > 0$, $f(u)$ is continuous, and has continuous derivatives for all values, $-\infty < u < \infty$. The coefficients of terms in these expansions are reduced from integrals, of multiplicity equal to the sample size, to products of coefficients, common to all populations of the class defined above, and integrals of no greater multiplicity than the number of groups of observations in the sample. Selected values of the common coefficients are given as well as illustrative examples for the Cauchy and "squared hyperbolic secant" population.

32. Note on the Behavior of the Characteristic Function of a Random Variable at Zero. M. ROSENBLATT, University of Chicago.

Let X be a random variable with characteristic function $\phi(z)$. Let $X_n = X$ when $|X| < n^{1/\alpha}$ and let $X_n = 0$ when $|X| \geq n^{1/\alpha}$. The following theorems are proved: (1) $1 - \phi(z) = o(|z|^\alpha)$, $0 < \alpha < 1$, at $z = 0$ if and only if $n \cdot \Pr(|X| > n^{1/\alpha}) = o(1)$. (2) $1 - \phi(z) = o(|z|^\alpha)$, $1 \leq \alpha < 2$, at $z = 0$ if and only if $n \cdot \Pr(|X| > n^{1/\alpha}) = o(1)$ and $E(X_n) = o(1)$. The results are obtained by making use of W. Feller's necessary and sufficient conditions for the weak law of large numbers (see W. FELLER, *Acta Univ. Szeged*, Vol. 8 (1937), pp. 191-201).

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest.

Personal Items

Dr. R. R. Bahadur, who received his Ph.D. in mathematical statistics from the University of North Carolina in June, 1950, is now an instructor in the Committee on Statistics of the University of Chicago.

Dr. T. A. Bancroft, Associate Professor of Statistics, Iowa State College, has been appointed Head of the Department of Statistics and Director of the Statistical Laboratory at Iowa State College.