

For small n these equations can be solved by iteration, which was done in constructing Table 1. Initial conditions are $U_k(0) = 1$, $U_i(0) = 0$ for $i \neq k$. It might be noted that the $U_i(j+1)$ are subtotals of the $U_i(j)$ so that the iteration proceeds very rapidly on an adding machine. The probability that $d \leq k/n$ is $[U_0(n) + U_1(n) + U_2(n) \cdots + U_k(n)]n!n!/(2n)!$.

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A NOTE ON THE SURPRISE INDEX

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Let $p_m (m = 0, 1, 2, \dots)$ be a set of probabilities of events E_m , and suppose that the event E_i , with probability p_i , actually occurred. Is the fact that E_i occurred to be regarded as surprising? In a recent article [1] this question is answered by introducing the surprise index S_i ,

$$(1) \quad S_i = (\Sigma p_m^2)/p_i,$$

which gives a comparison between the probability expected and that actually observed.¹ The event is to be regarded as surprising when S_i is large.

The author remarks on the difficulty of computing (1) for the Poisson and binomial distribution. The problem consists in evaluating the numerator, which we shall express here in terms of tabulated functions. The Poisson case leads to Bessel functions, the binomial case to Legendre or hypergeometric functions, and the asymptotic behavior involves square roots only.

1. *The Poisson case.* For the Poisson case we have $p_m = \lambda^m e^{-\lambda}/m!$ so that the generating function is

$$(2) \quad e^{-\lambda} e^{\lambda x} = \Sigma p_m x^m.$$

Let $x = e^{i\theta}$, then $e^{-i\theta}$; multiply; integrate from 0 to 2π ; and simplify slightly to obtain

$$(3) \quad \Sigma p_m^2 = (e^{-2\lambda}/\pi) \int_0^\pi e^{2\lambda \cos \theta} d\theta.$$

¹ Cf. also [6].