ABSTRACTS

ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Santa Monica meeting of the Institute, June 15 and 16, 1951*)

1. First Passage in Random Walks. T. E. Harris, The Rand Corporation.

Consider a random walk on the integers with transition probabilities p_r for $r \to r + 1$ and $q_r = 1 - p_r$ for $r \to r - 1$. Let N_{ij} be the first passage time from i to j. Explicit expressions are given for $P(N_{ij} < \infty)$ and $E(N_{ij} \mid N_{ij} < \infty)$, the latter expression sometimes being finite in transient chains. The second moment of N_{ij} can also be found. Sufficient conditions can be given for the limiting distribution of N_{0j}, as $j \to \infty$, to be exponential. The conditions include some walks with infinite mean recurrence times.

2. Distinct Hypotheses and Convex Sets. Lucien M. Le Cam, University of California, Berkeley.

Given a measure μ on a set $\mathfrak{X}$, the densities of probability with respect to μ on $\mathfrak{X}$ are considered as points in a Banach space B. A test is a point in the unit sphere of the conjugate Banach space. The distinctness of two hypotheses can be discussed using their convex hulls in B. Theorems by Mazur, Klee, etc., give necessary and sufficient conditions for distinctness. It is shown that the theorem by A. Berger and A. Wald corresponds to the case in which the closed convex hulls are disjoint. The result can thus be slightly extended.

3. Uniform Convergence of Random Functions with Applications to Statistics. Herman Rubin, Stanford University.

Let $X_1, \cdots, X_n, \cdots$ be a sequence of independent and identically distributed variables with values in an arbitrary space X. Let T be a compact topological space, and let $f(t, x)$ be a complex-valued function on $T + X$, measurable in x for each $t \varepsilon T$. Let P be the common distribution of the X_i. Then if there is an integrable g such that $| f(t, x) | \leq g(x)$ for all $t \varepsilon T$ and $x \varepsilon X$, and if there is a sequence S_i of measurable sets such that $P(X - \bigcup_{i=1}^{\infty} S_i) = 0$ and for each i, $f(t, x)$ is equicontinuous in t for $x \varepsilon S_i$, then with probability one, $(1/n) \sum_{k=1}^{n} f(t, X_k) \to \int f(t, x)\, dP(x)$ uniformly for $t \varepsilon T$, and the limit of the function is continuous. Since $f(t, x) = e^{itx}$ satisfies the conditions of the theorem, the sample characteristic function converges to the population characteristic function uniformly with probability one in any bounded interval. Log $L(x \mid \theta) = f(x, \theta)$ satisfies the conditions of the theorem for many distributions, including the multivariate normal, Poisson, Cauchy, χ^2, and double exponential, and hence the almost certain convergence of maximum likelihood estimates to the true values if the parameter is restricted to a compact set is established for those cases. More difficult estimation procedures can also be shown to be consistent by this method.

4. A Sequential Test for Linear Hypotheses. Paul G. Hoel, University of California, Los Angeles.

A sequential test for the general linear hypothesis is obtained by employing methods similar to those introduced by Wald in deriving his sequential t test. Optimum properties of the test are studied. An explicit expression for p_{1m}/p_{0m} is obtained, the evaluation of which requires incomplete Gamma function tables.