To prove (a), note that for $|t_i| < \alpha_i$, $x_1 \geq 0$, we have

$$(4)\qquad \begin{aligned} &\iint_{u_i \geq x_i} dF_n(u_1, u_2) \leq \exp(-|t_1|x_1 - |t_2|x_2) \\ &\iint_{u_i \geq x_i} \exp(|t_1|u_1 + |t_2|u_2)\, dF_n(u_1, u_2) \leq M_0 \exp(-|t_1|x_1 - t_2|x_2), \end{aligned}$$

where $\varphi_n(|t_1|, |t_2|) \leq M_0$. Such a number $M_0 = M_0(t_1, t_2)$ exists since $\{\varphi_n(|t_1|, |t_2|)\}$ converges for $|t_i| < \alpha_i$. This gives an estimate for $M_n(x_1, x_2)$, which shows that (a) holds. The Helly selection principle ([2], pp. 60–62 and 83) leads to (b). The relations (c) and (d) follow immediately from Theorem 1.

From Theorems 1 and 2 we obtain

THEOREM 3. *Let $\{F_n(x_1, x_2)\}$ be a sequence of df's and let $\{\varphi_n(t_1, t_2)\}$ be the corresponding sequence of mgf's which are all assumed to exist for $|t_i| < \alpha_i$. Then the necessary and sufficient condition for the convergence of $\{\varphi_n(t_1, t_2)\}$ for $|t_i| < \alpha_i$ is that the relations* (a) *and* (b) *of Theorem* 2 *be satisfied.*

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A NOTE ON THE MAXIMUM VALUE OF KURTOSIS

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In "A note on skewness and kurtosis," J. E. Wilkins (*Annals of Math. Stat.* Vol. 15 (1944), pp. 333–335) gave a short and elegant proof of the inequality for skewness and kurtosis

$$(1)\qquad \beta_2 \geq \beta_1^2 + 1.$$

Then he gave an upper bound, depending on the population size N, for the skewness:

$$(2)\qquad \max \beta_1 = (N - 2)/(N - 1)^{\frac{1}{2}}.$$

Now we shall derive an upper bound for the kurtosis. It will appear that the sign "=" in (1) is valid for the upper bounds, and the two maximum values indeed arise in the same "extreme" population.

To find the maximum value of the kurtosis β_2 we consider the function Σx_i^4 in the x-space, where $\Sigma x_i^2 = N$ and $\Sigma x_i = 0$. We have to maximize $\Sigma x_i^4 - \lambda \Sigma x_i^2 - \mu \Sigma x_i$. The maximizing values are given by the N equations, found by differentiation with respect to x_i

$$(3)\qquad 4x_i^3 - 2\lambda x_i - \mu = 0,$$

[1] "Aspirant" of the Belgian National Foundation for Scientific Research.