To prove this let u be a vector $(u_1, \cdots, u_n)$. Let U be the set of u such that the property described in (4) holds. We have

$$P[u \varepsilon U] = n! \int_U \prod_{j=1}^{n} dF[u_j, f(u_j)]. \tag{5}$$

Let $z_j = F[u_j, f(u_j)]$, $z = (z_1, \cdots, z_n)$

$$P[u \varepsilon U] = n! \int_Z dz, \tag{6}$$

where

$$z \varepsilon Z \quad \text{if} \quad \max_j |z_j - j/n| < \lambda \quad \text{and} \quad \max_j |z_j - (j+1)/n| < \lambda.$$

Since (6) does not depend on $F(x, y)$, the probability is the same for all $F(x, y)$ with the given properties. Nor does (6) depend upon the particular choice of $f(u)$.

The expression (5) is the probability distribution of the type (1) for the single-variable distribution $F[x, f(x)]$. We can test the hypothesis that a given random sample was derived from a particular distribution by means of the maximum deviation of the distribution from the step function derived from the sample. Values of the probabilities have been tabulated by Massey [1].

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ON THE NECESSARY AND SUFFICIENT CONDITIONS FOR THE CONVERGENCE OF A SEQUENCE OF MOMENT GENERATING FUNCTIONS

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In a previous paper ([1], pp. 61–69) the author studied the reciprocal relation between the convergence of a sequence of df's (distribution functions) and the convergence of the corresponding sequence of mgf's (moment generating functions) in the univariate case. It is the purpose of the present paper to give necessary and sufficient conditions for the convergence of a sequence $\{\varphi_n(t_1, t_2)\}$ of mgf's in two dimensions. The results can be extended to Euclidean spaces of higher dimensions.