That this is possible is most easily seen geometrically by observing that the line $\pi' = \pi''$ separates point 1 from points 2 and 3, so that there exist weights p_{10}, p_{20}, p_{30} for points 1, 2, 3, respectively, so that the center of gravity of 1 and 2 lies on the line $\pi' = \pi''$, as does that of 1 and 3. Also the center of gravity of these three points with the assigned weights lies on the same side of $\pi' = \pi''$ as 2 and 3 while 4 lies on the opposite side. Thus we can determine p_{40} so that

$$\sum_{i=1}^{4} p_{i0}\pi'_{i0} = \sum_{i=1}^{4} p_{i0}\pi''_{i0}.$$

Finally we take

$$(2) \qquad p_i = \frac{p_{i0}}{\sum_{j=1}^{4} p_{j0}}, \qquad \pi'_i = \frac{\pi'_{i0}}{\sum_{j=1}^{4} p_j \pi'_{j0}}, \qquad \pi''_i = \frac{\pi''_{i0}}{\sum_{j=1}^{4} p_j \pi''_{j0}}.$$

Then all the conditions (a), (b), (c), (d) are satisfied. By similar reasoning it is easy to see that the parameters can be chosen so that w_1 and w_2 have arbitrary sizes α_1 and α_2, respectively.

It is possible to obtain cases where H_0 contains a continuum of simple hypotheses, for example

$$H_0(\lambda): P\{X = i\} = \lambda p'_i + (1 - \lambda)p''_i,$$

with $0 \leqq \lambda \leqq 1$, where p'_i, p''_i are obtained as in the main part of this paper. The same tests are most powerful and similar. Many interesting questions arise but they seem not to be of any real statistical importance.

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NOTE ON THE ESTIMATION OF A BIVARIATE DISTRIBUTION FUNCTION

By Paul B. Simpson

University of Oregon

A continuous cumulative probability distribution $F(x)$ can be estimated from a random sample (x_i), $i = 1, \cdots, n$, by the step function $G(x) = j/n$, where j is the number of $x_i \leq x$. In this single variable case, it is known that the probability distribution

$$(1) \qquad P\{\max_x |F(x) - G(x)| < \lambda\}$$