ON THE DISTRIBUTION OF AN ANALOGUE OF STUDENT'S t

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1. Introduction. The independence of the sample range and the mean in random samples from a normal population has been already established [1], [2]. Using this property of independence, J. F. Daly [1] has shown that with the help of the distribution law of the sample range, w, tabulated by Pearson and Hartley [3] the probability distribution of an analogue of Student's t-test given by $G = (\bar{x} - a)/w$ can be studied, where $\bar{x}$ is the mean and a the location parameter. E. Lord [2] has prepared, by quadrature, exhaustive tables of levels of significance of G for sample size varying from 2 to 20 corresponding to the probabilities 0.10, 0.05, 0.02, 0.01, 0.002, and 0.001. An approximation to the distribution of u = const. G has been studied by P. B. Patnaik [4] and the power of the u-test has been investigated by Lord [5]. E. S. Pearson [6] has examined the effect of nonnormality on u-test involving range. The purpose of the present note is to develop the probability distribution of G as a series whose terms reduce to Beta functions and to observe the efficiency of the G-test.

2. The distribution of G. For a sample of size n from a normal population with mean a and standard deviation unity the distribution of $\bar{x}$ is given by

$$p(\bar{x}) = \frac{\sqrt{n}}{\sqrt{2\pi}} e^{-\frac{1}{2}n(\bar{x}-a)^2}, \tag{1}$$

and the distribution of the semi-range, W, has been shown by the author [7] to be given by

$$p(W) = ke^{-(n+4)W^2/6} \sum_{i=0}^{\infty} C_i W^{n+2i-2}, \tag{2}$$

where

$$k = \frac{n-1}{2^4\Gamma(5)n^{7/2}} (\sqrt{2/\pi})^{n-1} \tag{3}$$

and C_i are functions of n. The distribution (2) is useful for small sample sizes, and C_i coefficients have been computed for sample sizes up to 8, using an appropriate expansion [8] of the normal probability integral.

Since $\bar{x}$ and W are independently distributed,

$$p(\bar{x}, W) = \sqrt{\frac{n}{2\pi}}\, ke^{-\frac{1}{2}n(\bar{x}-a)^2} e^{-(n+4)W^2/6} \sum_{i=0}^{\infty} C_i W^{n+2i-2}. \tag{4}$$

Putting

$$G = \frac{\bar{x} - a}{2W}, \tag{5}$$

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