A MULTIVARIATE GAMMA-TYPE DISTRIBUTION[1]

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Introduction. Mehler has shown that the two-variate probability density function (pdf) for correlated variates, each of which has a marginal Guassian distribution, can be expressed as a series bilinear in Hermite polynomials:

$$\frac{1}{2\pi\sqrt{1-\rho^2}} \exp\{-\tfrac{1}{2}(x^2 - 2\rho xy + y^2)/(1-\rho^2)\}$$
$$= \frac{1}{2\pi} \exp\{-\tfrac{1}{2}(x^2+y^2)\}\left[1 + \rho H_1(x)H_1(y) + \frac{\rho^2}{2!} H_2(x)H_2(y) + \cdots\right].$$

Kibble [5] has extended this result to any number of variables and noticed a small difference between the generalization and the particular case due to Mehler.

It is known that Mehler's series is not an isolated result, there being a similar series bilinear in Laguerre polynomials, discussed by Hardy [3], Watson [6], and Kibble [4], and series bilinear in certain other other polynomials, discussed by Campbell [2], and by Aitken and Gonin [1]. All these results can be generalized for any number of variables in much the same way as Kibble has generalized Mehler's result. These generalizations are contained in Krishnamoorthy's thesis "Multivariate Distribution Functions" (in the library of the University of Madras). In the present paper the generalization involving Laguerre polynomials is given.

1. Notation and summary. It was shown by Kibble [4] that a two-variate distribution function, in which each of the variates x_i, $i = 1, 2$, has the frequncy function

$$\phi(x_i) = \frac{x_i^{p-1} e^{-x_i}}{\Gamma(p)}, \tag{1.1}$$

may be represented by

$$\phi(x_1)\phi(x_2)\left[1 + \frac{\rho^2}{p} L_1(x_1, p)L_1(x_2, p) + \frac{\rho^4}{2!p(p+1)} L_2(x_1, p)L_2(x_2, p) + \cdots\right],$$

where $L_r(x, p)$, $p > 0$, is the generalized Laguerre polynomial of degree r satisfying

$$L_r(x, p) \equiv r! L_r^{(p-1)}(x) = \frac{\left(-\frac{d}{dx}\right)^r [x^r\phi(x)]}{\phi(x)}. \tag{1.2}$$

[1] Sections 1 to 4 of this paper, deriving the distributions, were written by the first author; Section 5, on the convergence of certain series, was contributed by the second author.