# THE FORMATIVE YEARS OF ABRAHAM WALD AND HIS WORK IN GEOMETRY

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In the fall of 1927, a man of 25 called at the Mathematical Institute of the University of Vienna. Since he expressed a predilection for geometry he was referred to me. He introduced himself as Abraham Wald. In fluent German, but with an unmistakable Hungarian accent, Wald explained that he had carried on most of his studies at the elementary and secondary school levels at home, mainly under the direction of his older brother Martin, a capable electrical engineer in Cluj (Kolozsvár, Klausenburg). He had just arrived in Vienna in order to study mathematics at the university. Geometry had interested him ever since he was fourteen. More recently he had been reading Hilbert's "Grundlagen der Geometrie" (Foundations of Geometry) and he saw possibilities for improving these foundations by omitting some postulates and weakening others. I suggested to Wald that he write up his results {5}[1] (one of his proofs was later incorporated into the seventh edition of Hilbert's book) and at the same time recommended some additional reading.

Wald enrolled in the university, but during the next two years Vienna did not see much of him. The system of complete freedom which at that time prevailed in the universities of Central Europe—a detrimental system for weak students—kept the gifted ones from wasting semesters on courses the content of which they could absorb in a few weeks of concentrated reading. Moreover, Wald had to serve in the Rumanian army.

It was not until February 1930 that he and I again had extended conversations. Then he came unexpectedly to hand me a manuscript which purported to contain the solution of a famous problem. It was a serious piece of work, but an error at the very end invalidated the result. Wald was visibly disappointed. But a few days later he returned to tell me that, during the last week, he had been sitting in on my lectures on metric geometry—the first university lectures he ever attended—and that he planned to follow this entire course. Moreover, he wanted to try his hand at some problem in this field. I had just introduced the "between" relation in metric spaces: The point $q$ is between the points $p$ and $r$ if, and only if, $p \neq q \neq r$ and the three distances between the points satisfy the equality

$$d(p, q) + d(q, r) = d(p, r).$$

I asked Wald whether he would like to try to characterize this "betweenness" among the ternary relations in a metric space. Four weeks later he brought me

[1] References are listed in "The publications of Abraham Wald," pp. 29–33 of this issue.