The function $K(x, y)$ is an increasing function of x and an increasing function of y, provided $x + y < 1$. Conditions (1) and (2) imply that $1 - L(\theta_0) \leqq \alpha$, $L(\theta_1) \leqq \beta$. Hence if $\alpha + \beta < 1$, we have

$$K(1 - L(\theta_0), L(\theta_1)) \leqq K(\alpha, \beta). \tag{15}$$

Inequality (4) now follows from relations (12) to (15).

Concerning the conditions for equality, it suffices to observe that in (10) the sign of equality holds if and only if there exist constants C_0 and C_1 such that

$$P_\theta\left\{\prod_{j=1}^{n} \frac{f(X_j, \theta)}{f(X_j, \theta')} = C_i \mid S \text{ accepts } H_i\right\} = 1, \qquad i = 0, 1,$$

where the usual notation for conditional probabilities is used. This can be verified from Wald's proof. The conditions for equality in (12), (13), (15) are obvious.

REFERENCES

[1] J. V. Uspensky, *Introduction to Mathematical Probability*, McGraw-Hill Book Co., New York and London, 1937.
[2] A. Wald, "Sequential tests of statistical hypotheses," *Ann. Math. Stat.*, Vol. 16 (1945), pp. 117–186.

SOME INEQUALITIES ON MILL'S RATIO AND RELATED FUNCTIONS

By M. R. Sampford
University of Oxford

1. Introduction. Mill's ratio is defined as

$$R_x = e^{\frac{1}{2}x^2} \int_x^\infty e^{-\frac{1}{2}u^2}\, du. \tag{1}$$

Gordon [1] and Birnbaum [2] have given, respectively, upper and lower limits for R_x as

$$\tfrac{1}{2}\{\sqrt{4 + x^2} - x\} < R_x < 1/x, \qquad x > 0. \tag{2}$$

Murty [3] has shown how limits to R_x of any required degree of accuracy can be derived for $x > 0$ by the use of successive convergents of Laplace's expression for the normal integral as a continued fraction. No limits have, as yet, been published for $x < 0$.

If the functions $\nu(x)$ and $\lambda(x)$ are defined by $\nu(x) = 1/R_x$, $\lambda(x) = \nu'(x) = \nu(\nu - x)$, the inequalities

$$0 < \lambda < 1, \tag{3}$$

$$\lambda' = \nu\{(\nu - x)(2\nu - x) - 1\} > 0 \tag{4}$$

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