ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Washington meeting of the Institute

April 29–May 1, 1953

1. Optimum Sample Sizes for Choosing the Largest of (k + 1) Means Using Minimax Methods. PAUL N. SOMERVILLE, University of North Carolina.

Assume we have $(k + 1)$ normally distributed populations with unknown means $a_0 \geqq a_i \geqq \cdots \geqq a_k$. It is decided to choose N individuals from these populations in such a way that the expected value of their total is as large as possible. A preliminary sample of n is taken from each population with the object of deciding from which population the further sample of size N should be taken. $N(a_i - a_0)$ is then the loss involved in choosing the population with parameter a_i. Assume the cost of the sample is a linear function of the sample size. Using results previously given it is shown that the minimax n is proportional to $N^{(2/3)}$. Explicit results are given for $k = 1, 2, 3, 4, 5$, for a one-stage preliminary sample. For the case $k = 2$, results for a two-stage sample are given. In the first stage, samples of n_1 are taken for each of the three populations. In the second stage, samples of n_2 are taken from each of the two populations with the largest means in the first stage. If $3n_1 + 2n_2 = 3n$, then it is found that the maximum expected loss is less for the two-stage sample than for the one-stage sample provided n_1/n_2 is greater than .37 (approximately). The optimum ratio in this sense is found to be $n_1/n_2 = 1.2$ (approximately). If for $n_1/n_2 = 1.2$, the maximum expected losses are equated by a reduction in the total preliminary sample size, a saving of 6.6 per cent over the one-stage procedure in the preliminary sample size is effected.

2. The Correspondence Between Two Classes of Balanced Incomplete Block Designs. W. S. CONNOR, National Bureau of Standards.

Let $\Sigma_1(n)$ denote the problem of constructing the design with parameters $v = \frac{1}{2}n(n + 1)$, $b = \frac{1}{2}(n + 1)\ (n + 2)$, $k = n$, $r = n + 2$, and $\lambda = 2$; and let $\Sigma_2(n)$ denote the problem of constructing the design with parameters $v = b = \frac{1}{2}(n + 1)\ (n + 2) + 1$, $r = k = n + 2$, and $\lambda = 2$, $(n > 1)$. It is shown that $\Sigma_1(n)$ has a solution only if $\Sigma_2(n)$ has a solution.

3. A Finite Frequency Theory of Probability. A. H. COPELAND, SR., University of Michigan.

This paper develops a new theory of probability, the finite frequency theory, in which probabilities are regarded as physical hypotheses. Associated with each probability is a system of predictions which can be tested by experiment. An experiment may either confirm or disagree with a given prediction. This theory of probability produces some complications in formal logic. However the theory and its associated deductive and inductive logics are in better agreement with modern scientific reasoning than the conventional probability theories and the conventional logics.

4. Characterizations of Complete Classes of Tests of Some Multiparametric Hypotheses, with Applications to Likelihood Ratio Tests. ALLAN BIRNBAUM, Columbia University.

Let H_0 be a simple hypothesis on a density function of the form

$$p_\varphi(e) = \exp\ \{\varphi_0 + \Sigma_1^k \varphi_i t_i(e) + t_0(e)\}.$$

Let T, the range of the sufficient statistic $t = (t_1, \cdots, t_k)$, be independent of t. Let V' be the class of nonrandomized decision functions $\delta(t)$ such that each $\delta(t) = 0$ just on the