SOME TESTS BASED ON ORDERED OBSERVATIONS FROM TWO EXPONENTIAL POPULATIONS[1]

By Benjamin Epstein and Chia Kuei Tsao

Wayne University

1. Introduction. Let $x_{11} \leqq x_{12} \leqq \cdots \leqq x_{1n_1}$, and $x_{21} \leqq x_{22} \leqq \cdots \leqq x_{2n_2}$, be two random samples (S_{n_1} and S_{n_2}) from populations having p.d.f.'s $f(x; A_1, \theta_1)$ and $f(x; A_2, \theta_2)$ respectively, where

$$f(x; A, \theta) = \frac{1}{\theta} \exp [-(x - A)/\theta]. \tag{1}$$

Let S_{r_1} and S_{r_2} be the sets of the first r_1 and r_2 smallest observations of S_{n_1} and S_{n_2} respectively. Then the p.d.f.'s of S_{r_1} and S_{r_2} are given, say, by

$$g(x_{11}, \cdots, x_{1r_1}; A_1, \theta_1) \quad \text{and} \quad g(x_{21}, \cdots, x_{2r_2}; A_2, \theta_2),$$

where

$$g(x_1, x_2, \cdots, x_r; A, \theta) = \frac{n!}{(n-r)!} \frac{1}{\theta^r} \exp\left\{-\frac{1}{\theta}\left[\sum_{i=1}^{r} (x_i - A) + (n-r)(x_r - A)\right]\right\}. \tag{2}$$

The likelihood ratio tests based on the complete sets, S_{n_1} and S_{n_2} are special cases of those obtained by Sukhatme [2], [3]. It can be shown that similar likelihood ratio tests based on S_{r_1} and S_{r_2} may be obtained by following Sukhatme's procedure [2]. In this paper these likelihood ratio tests are reduced to equivalent tests which are expressed in terms of the well known chi square and Snedecor's F distributions. Furthermore, some of the tests obtained in this paper can be extended to k-sample tests.

Since percentage points for χ^2 and F distributions are tabled, tests involving these random variables are useful in applications. We remark that the likelihood ratio test for the hypothesis H_5 (see Section 3) has been obtained by Paulson [1].

The results of this paper can be used in the field of life testing. A characteristic feature of such tests is that observations become available in order of magnitude. The assumption of an exponential distribution of life is a reasonable one to make in some applications (e.g., electron tube life). The parameter A can be interpreted as minimum life (also called sensitivity limit in fatigue failure problems) and the parameter θ is the mean life measured from A as a starting point. From the life test point of view one has a sample of size n_1 from population 1 and a sample of size n_2 from population 2, the two populations one wishes to compare. Procedures

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