[2] W. G. COCHRAN, "The distribution of the largest of a set of estimated variances as a fraction of their total," *Ann. Eugenics*, Vol. 11 (1941), pp. 47–52.
[3] EISENHART, HASTAY, AND WALLIS (Editors), *Selected Techniques of Statistical Analysis*, McGraw-Hill Book Co., 1947, pp. 390–391.
[4] A. WALD, *Statistical Decision Functions*, John Wiley and Sons, 1950, pp. 123–130.

AN EXTENSION OF THE BUFFON NEEDLE PROBLEM

BY NATHAN MANTEL

Biometrics Section, National Cancer Institute[1]
Bethesda, Maryland

1. Introduction. An empirical determination of the value of π can be made from the relationship[2]

$$E = 2L_1L_2/(\pi A), \tag{1}$$

where E is the expected number of intersections of a group of line segments of total length L_1 with a group of line segments of total length L_2, both groups being distributed over an area A. This relationship applies under the following conditions.

(i) The arrangement of the two groups of line segments on the area A must be independent of each other, but the individual line segments of a group may have a systematic arrangement relative to each other.

(ii) The arrangement of at least one of the two groups of line segments on the area A must be at random. The randomness must be such that the probability of a specified point on a line segment falling into a sub-area of A is proportional to its area and the segment may assume any angle relative to some base line with equal probability.

Two applications of this relationship to the estimation of π are considered below.

2. The Buffon needle problem using a parallel line system. Consider an area A on which is superimposed a series of equally spaced parallel lines (without loss of generality we shall take the common distance between them to be unity), on which a straight line of length $L \leqq 1$ is allowed to fall at random. At each fall the line must either intersect the series of parallel lines only once, or not at all. Thus the expected number of intersections, E, is the probability, P, of an intersection occurring at a fall. And since for this system the total length of the

Received 6/12/52, revised 12/22/52.

[1] National Institutes of Health, Public Health Service, Department of Health, Education and Welfare.

[2] This relationship is developed in passing by Cornfield and Chalkley, "A problem in geometric probability," *J. Wash. Acad. Sci.*, July, 1951.