spaces in order to construct orthogonal arrays. In particular, this method enabled them to construct an orthogonal array (81, 10, 3, 3). They proved, on the other hand, that in the case considered the maximum number of constraints cannot exceed 12 (Theorem 2C). Hence they state: "We do not know whether we can get 11 or 12 constraints in any other way." It is shown that no such way exists.

26. Some Contributions to the Theory of Markov Chains. (Preliminary Report.) CYRUS DERMAN, Columbia University.

Suppose that a collection of particles are moving about independently according to probabilities given by a Markov chain with transition matrix $P = \{p_{ij}\}$ $i, j, = 0, 1, \cdots$. Let $A_n(i)$ denote the number of particles in state i at time n $(i, n = 0, 1, \cdots)$. Some sufficient conditions on P and on the distributions of the $A_0(i)$'s were found such that for any set $i_1, \cdots, i_r$, the joint distribution of $A_n(i_1), \cdots, A_n(i_r)$ tends as $n \to \infty$ to that of r independent Poisson distributions. Consider a recurrent Markov chain $\{X_n\}$. Let $N_n(i) = \{$the number of r's such that $X_r = i$ for $1 \leqq r \leqq n\}$ $i = 0, 1, \cdots$. The following theorem was proved. If $H(n)$ is any nondecreasing unbounded function and if i and j are any two states belonging to the same class, then with probability one the inequality $| (N_n(j) - a_{ij}N_n(i))/(N_n(i) + N_n(j))^{\frac{1}{2}}H(N_n(i) + N_n(j)) | > b_{ij}$, where a_{ij} and b_{ij} are certain constants, will be satisfied for infinitely many or at most finitely many n according as $\sum_{n=1}^{\infty} H(n) \exp(-H^2(n)/2)/n$ diverges or converges. Sufficient conditions were given such that for any $i_1, \cdots, i_r$ the distribution of $N_n(i_1), \cdots, N_n(i_r)$ properly normalized approaches a multivariate normal distribution. The asymptotic covariance matrix was computed.

27. Minimax Invariant Procedures for Estimating Cumulative Distribution Functions. OM P. AGGARWAL, University of Washington.

Let $x_1 < x_2 < \cdots < x_n$ be the ordered observations on a chance variable with cumulative distribution function F. Let $\hat{F}$ denote an estimate of F based only upon the sample. The minimax invariant procedures of estimating F are obtained for two classes of loss functions $L(F, \hat{F})$. For $L(F, \hat{F}) = \int_{-\infty}^{\infty} | F(x) - F(x) |^r dx$, with integer $r \geqq 1$, the minimax invariant procedure is to estimate F by a step function $\hat{F}(x) = c_j$ for $x_j \leqq x < x_{j+1}$; $j = 0, 1, \cdots n$, where x_0 and x_{n+1} denote $-\infty$ and $+\infty$ and c_j is obtained as the root of an equation of degree $(n + r)$ when r is odd and of degree $(r - 1)$ when r is even. For the special case $r = 1$ the value of c_j is the median of a Beta distribution. For $r = 2$, one obtains $c_j = (j + 1)/(n + 2)$. For the class of loss functions $L(F, \hat{F}) = \int_{-\infty}^{\infty} [F(x) - \hat{F}(x)]^{2k}/F(x)[1 - F(x)] dx$, one again obtains for minimax invariant procedure step functions with c_j determined as root of an equation of degree $(2k - 1)$. In particular for $k = 1$ this optimum procedure turns out to be the usual sample cumulative function with $c_j = j/n$. (Work supported by the Office of Naval Research.)

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest

Personal Items

F. J. Anscombe of Cambridge, England has been appointed Research Associate in the Department of Mathematics, Princeton University, for the year 1953–1954.