(4.3) $$\sum_{n=1}^{\infty} a_n = \infty, \qquad \sum_{n=1}^{\infty} a_n^2 < \infty.$$

Define a recursive approximation scheme as follows. Let x_1 be arbitrary and define

(4.4) $$x_{n+1} = x_n + a_n z_n$$

where $z_n = +1$ if $y_n \leqq \alpha$ and $z_n = -1$ if $y_n > \alpha$, and y_n is a random variable distributed according to $H(y \mid x_n)$. Then, by applying Theorem 1 with $\alpha = 0$ and $y_n = -z_n$, we obtain

THEOREM 3. *If conditions* (4.1), (4.2), *and* (4.3) *hold, then* $P\{\lim x_n = \theta\} = 1$.

I should like to thank Mr. Lucien LeCam for many helpful discussions concerning this problem. I should also like to thank the referee for pointing out that the condition of uniform boundedness of $M(x)$ in Section 2 could be replaced by the present condition (2.1).

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A NOTE ON THE ROBBINS-MONRO STOCHASTIC APPROXIMATION METHOD[1]

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Introduction. The almost certain convergence of the RM process and related stochastic approximation procedures is proved by Blum [1] in a paper appearing elsewhere in this issue. In the present note we consider the method originally proposed by Robbins and Monro [2] with a further restriction on the constants a_n. Our aim is to obtain, by elementary methods, an estimate of the order of magnitude of $b_n = E(x_n - \theta)^2$. This estimate is sharp enough to enable us to prove strong convergence for certain types of sequences a_n. The method adopted in [1], while being more general, does not yield information about the behavior

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