A NOTE ON PARTIALLY BALANCED DESIGNS

By Marvin Zelen

National Bureau of Standards

It is well known [1], [2] that a singular group divisible design containing two associate classes can be derived from a balanced incomplete block design by replacing each treatment by n treatments. In this paper it is shown that a partially balanced design with $(m + 1)$ associate classes can be derived from a partially balanced design with m associate classes by replacing each treatment by n treatments.

The definition of a partially balanced incomplete block design with m associate classes can briefly be described as an experimental plan

(i) having v treatments arranged in b blocks such that each block contains k experimental units,

(ii) where each treatment is replicated r times and no treatment occurs more than once in any block,

(iii) such that with respect to any treatment t, the remaining treatments can be divided into m associate classes such that the ith class contains n_i treatments and t occurs in λ_i blocks with each of the treatments in the ith class $(i = 1, 2, \cdots, m)$,

(iv) and if two treatments are kth associates, the number of treatments common to the ith associates of one and the jth associates of the other treatment is p_{ij}^k (for $i, j, k = 1, 2, \cdots, m$, with $p_{ij}^k = p_{ji}^k$), and is independent of the particular pair of treatments.

It has been shown [3] that the following relations hold between the parameters of the design.

$$bk = vr \tag{1}$$

$$\sum_{i=1}^{m} n_i = v - 1 \tag{2}$$

$$\sum_{i=1}^{m} n_i \lambda_i = r(k - 1) \tag{3}$$

$$\sum_{j=1}^{m} p_{ij}^k = \begin{cases} n_i, & \text{for } i \neq k \\ n_i - 1 & \text{for } i = k \end{cases} \tag{4}$$

$$n_i p_{jk}^i = n_j p_{ik}^j = n_k p_{ij}^k. \tag{5}$$

The main result of this paper can be stated in the following theorem.

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