We can now establish the desired result.

THEOREM 2. *If X and Y are independent observations on the same unimodal random variable, then $X - Y$ is unimodal.*

We prove the theorem in three parts.

PART I. If X has as possible values only finitely many integers, the theorem is an immediate consequence of the preceding one. The a's are taken to be the probabilities of the successive possible values of X. Since $P(X - Y = k) = S_k$ for k a positive integer, and since $X - Y$ has a distribution symmetric about 0, the theorem follows.

PART II. Let the possible values of X now be numbers of the form $r\Delta$, where $\Delta > 0$ and r is any integer. For simplicity we may assume 0 to be a mode. For every positive integer s, define

$$X_s' = \begin{cases} X \text{ if } |X| \leqq s, \\ 0 \text{ if } |X| > s, \end{cases} \qquad Y_s' = \begin{cases} Y \text{ if } |Y| \leqq s \\ 0 \text{ if } |Y| > s. \end{cases}$$

That $X_s' - Y_s'$ has a unimodal distribution is an immediate consequence of Part I. But since $P(X_s' - Y_s' \neq X - Y) \to 0$ as $s \to \infty$, we see that $X - Y$ must also have a unimodal distribution.

PART III. Now suppose X has a density f, with mode at m. For each positive integer s, define

$$X_s'' = [(X - m)\sqrt{s}]/\sqrt{s},$$

where $[u]$ denotes the greatest integer less than u. The cumulative distribution G_s'' of $X_s'' - Y_s''$ cannot ever differ from G by more than a quantity which tends to 0 as $s \to \infty$. However, G_s'' is unimodal, by Part II. If G were not unimodal, we could find $\epsilon > 0$, $\Delta > 0$, and $u - \Delta > 0$ such that $G(u - \Delta) + G(u + \Delta) + \epsilon < 2G(u)$, which would yield a contradiction.

REFERENCE

[1] K. L. CHUNG, "Sur les lois de probabilité unimodales," *C. R. Acad. Sci. Paris*, Vol. 236 (1953), pp. 583–584.

NOTE ON A THEOREM OF LIONEL WEISS[1]

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1. Introduction. In a recent paper [1] it was pointed out by Lionel Weiss that the class of sequential probability ratio tests is complete in a very strong sense. The purpose of the present note is to show how this result can be derived from a

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