From well known limit theorems,

9)
$$\lim_{\lambda\to\infty} A_\lambda = \lim_{\lambda\to\infty} \Pr\{X_\lambda \leqq \lambda\} = \lim_{\lambda\to\infty} \Pr\left\{\frac{X_\lambda - \lambda}{\sqrt{\lambda}} \leqq 0\right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-y^2/2}\, dy = \tfrac{1}{2}.$$

The second portion of inequality (2) follows directly from (8) and (9). The remainder is a consequence of these, of the fact that $A_{n,\lambda}$ decreases monotonically from $A_{n,n}$ to $A_{n,n+1}$ in the interval $[n, n+1)$, and of

10)
$$A_{n,n+1} > A_{0,1} = e^{-1}, \qquad n = 1, 2, \cdots.$$

If an additional term is included in the sum, we note that

11)
$$b_\lambda = \sum_{j=0}^{[\lambda]+1} \frac{\lambda^j e^{-\lambda}}{j!} \geqq \tfrac{1}{2} \qquad \text{for all } \lambda \geqq 0,$$

since $b_\lambda > A_{n+1,n+1}$ for $\lambda \leqq n + 1$. A final reformulation of part of (2) is

12)
$$\Pr\{X_\lambda \leqq \lambda\} > \Pr\{X_\lambda > \lambda\}, \qquad \text{all integral } \lambda > 0.$$

NOTE ON LINEAR HYPOTHESES WITH PRESCRIBED MATRIX OF NORMAL EQUATIONS

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The existence theorem proven in this note relates to the problem of finding an experimental design leading to the analysis determined by the given rational matrix A of the normal equations. The matrix B found by the method used in the proof always has an interpretation as specifying the rational values of some set of regression variables. In the interesting case in which the entries of A are integers, so are the entries of B, but B is not in general interpretable as an analysis of variance. The transpose of a matrix A will be denoted by A^T.

Theorem. *Let A be a symmetric positive semidefinite matrix with rational integral entries. There exist a rational integer a and a matrix B having rational integral entries such that $BB^T = a^2A$.*

Proof. There exists a nonsingular matrix P such that $P^TAP = D$, a diagonal matrix, where P and D have rational entries ([1], p. 56). Then $(P^T)^{-1}$ has rational entries. Let a_1 be the least common denominator of the entries of P^T, a_2 of $(P^T)^{-1}$, and a_3 of D. Let $a^{1/2} = a_1a_2a_3$. Then $a^{1/2}P^Ta^{1/2}AP = aD$. If A is positive semidefinite, then aD has only positive integers or zeros on its diagonal. Let B_1 be a $n \times 4n$ matrix composed of four diagonal $n \times n$ matrices placed side by side,

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