values of Y between the limits

$$L_1(X) = \tfrac{1}{2}\{2rX + rk^2 - k\sqrt{k^2r^2 + 4rX(1 + r)}\}, \quad \text{and}$$

$$L_2(X) = \tfrac{1}{2}\{2rX + rk^2 + k\sqrt{k^2r^2 + 4rX(1 + r)}\}.$$

Therefore, for these limits, we have that $P[L_1(X) \leqq Y \leqq L_2(X)]$ is approximately equal to α, no matter what the value of θ is.

REFERENCE

[1] J. NEYMAN, "Outline of a theory of statistical estimation based on the classical theory of probability," *Philos. Trans. Roy. Soc.* London, Ser. A, Vol. 236 (1937), pp. 333–380.

AN INCONSISTENCY OF THE METHOD OF MAXIMUM LIKELIHOOD

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An example was given by Neyman and Scott [2] to show that there are situations where the method of maximum likelihood leads to inconsistent estimators. In their example considered, the observations were supposed to be drawn from an infinite sequence of distinct populations involving an infinite sequence of nuisance parameters.

An example is given here to demonstrate that even in simple situations where all the observations are independently and identically distributed and involve only one unknown parameter, the method of maximum likelihood may lead us astray. The example typifies the situations where the correct method of setting up a point estimate should begin with a test of hypothesis between two composite alternatives.

Let A be the set of all rational numbers in the closed interval (0, 1) and B any countable set of irrational numbers in the same interval. Let X be a random variable that takes the two values 0 and 1 with

$$P(X = 1) = \begin{cases} \theta & \text{if } \theta \in A, \\ 1 - \theta & \text{if } \theta \in B. \end{cases}$$

If r is the total number of 1's in a set of n random observations on X, then from the rationality of r/n it follows at once that the maximum likelihood estimator of θ is r/n. But r/n converges (in probability) to θ or $1 - \theta$ according as $\theta \in A$ or $\theta \in B$.

Now, since A and B are both countable sets, it follows [1] that there exists a consistent test for the composite hypothesis $\theta \in A$ against the composite alterna-

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