follow. It may be that the use of decision limits which depend upon the value of n will produce a better test than (2), which uses the customary constant limits for the sequential probability ratio test.

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TWO COMMENTS ON "SUFFICIENCY AND STATISTICAL DECISION FUNCTIONS"

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In the following comments we employ the notation and definitions of [1]. The first comment answers a question raised in [1] by giving an example of a necessary and sufficient subfield which cannot be induced by a statistic. The second remark clarifies this example somewhat by discussing the connection between statistics and subfields in general. It was hoped that this connection would be so close as to provide the answer to another question raised in [1]: whether the existence of a necessary and sufficient subfield implies that of a necessary and sufficient statistic. However, an example given at the end of the second comment shows that such a result cannot be proved without making deeper use of sufficiency.

1. A counter example. The following result was communicated to us by David Blackwell.

Lemma 1. (*Blackwell*). *Let* $\mathbf{S}_0$ *be a proper subfield of* $\mathbf{S}$ *and suppose that for each* x *the set* $\{x\}$ *consisting of the single point* x *is in* $\mathbf{S}_0$. *Then* $\mathbf{S}_0$ *cannot be induced by a statistic.*

Proof. Suppose there exists such a statistic, say T, and let $\mathbf{T}$ be the field of sets B in the range of T such that $T^{-1}(B)\ \varepsilon\ \mathbf{S}$. Since $\{x\}\ \varepsilon\ \mathbf{S}_0$, there exists $B\ \varepsilon\ \mathbf{T}$ such that $T^{-1}(B) = \{x\}$, and, by definition of $\mathbf{T}$, a set $A\ \varepsilon\ \mathbf{S}$ such that $T(A) = B$. We therefore have $T^{-1}[T(A)] = \{x\}$, and since always $T^{-1}[T(A)] \supseteq A$, we have that $T^{-1}[T(x)] = x$ for all x. Therefore, if A is any set in $\mathbf{S}$, we see that $T^{-1}[T(A)] = A$ so that $A\ \varepsilon\ \mathbf{S}_0$ and hence our assumption that $\mathbf{S}_0$ is induced by T implies that $\mathbf{S}_0 = \mathbf{S}$.

We now give an example of a necessary and sufficient subfield that cannot be

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