NOTES

ON A SEQUENTIAL TEST FOR THE GENERAL LINEAR HYPOTHESIS[1]

By Paul G. Hoel

University of California, Los Angeles

1. Introduction. A few years ago I reported [1] on a sequential method for testing the general linear hypothesis, but held up publication until some of the properties of the method had been investigated further. Johnson [2] recently published a paper in which he obtained the same sequential test, but from an entirely different point of view. His method of derivation is based on showing, by means of a theorem of Cox [3], that the likelihood ratio approach to the problem can be used successfully. My method is a direct generalization of Wald's sequential t-test. This note outlines the nature of this generalization, and also points out some additional properties of the test.

2. Method. The general linear hypothesis assumes that the variables x_1, $x_2, \cdots, x_l$ with means $\mu_1, \mu_2, \cdots, \mu_l$ and common variance σ^2 possess the frequency function

$$f(x_1, \cdots, x_l) = (2\pi\sigma)^{-l} \exp\left\{\frac{-1}{2\sigma^2}\left[\sum_{i=1}^{k}(x_i - \mu_i)^2 + \sum_{i=k+1}^{l} x_i^2\right]\right\}. \tag{1}$$

It tests the hypothesis H_0: $\mu_1 = \cdots = \mu_p = 0$, for $p \leqq k$. The means $\mu_{k+1}, \cdots, \mu_l$ have the value zero. The parameters $\mu_{p+1}, \cdots, \mu_k$, and σ are nuisance parameters, and therefore make H_0 a composite hypothesis.

Following Wald's [4] procedure and notation for the sequential t-test, let the parameter space Ω be divided into the three regions ω_a, ω_r, and $\Omega - \omega_a - \omega_r$. The region ω_a will be chosen as that part of Ω where H_0 holds. The region ω_r will be chosen as that part of Ω where

$$\sum_{i=1}^{p} \frac{\mu_i^2}{\sigma^2} \geqq \lambda_0$$

where λ_0 is a selected constant. The boundary of ω_r will be denoted by S_r. As normalized weight functions choose

$$v_a(\theta) = \begin{cases} a_1 \sigma^{b_1} (2c)^{p-k}, & 0 \leqq \sigma \leqq c, |\mu_i| \leqq c, i = p+1, \cdots, k, \\ 0, & \text{elsewhere}; \end{cases}$$

$$v_r(\theta) = \begin{cases} a_2 \sigma^{b_2} (2c)^{p-k}, & 0 \leqq \sigma \leqq c, |\mu_i| \leqq c, i = p+1, \cdots, k, \\ 0, & \text{elsewhere}. \end{cases}$$

Received February 19, 1954.

[1] This research was sponsored by the O.N.R.