ESTIMATION OF THE MEAN AND STANDARD DEVIATION BY ORDER STATISTICS. PART II

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1. Introduction. In a previous paper [3], the best linear estimates of the mean and standard deviation for the rectangular, triangular, double exponential, and the exponential distributions were worked out. The best linear estimates were obtained by ranking the observations in ascending order and finding the best linear combination of them [2]. The variation of the coefficients in the estimates and the efficiencies of some other linear estimates were discussed.

This paper—which is a continuation of the previous one [3]—deals with three distributions: a U-shaped, a parabolic, and a skewed one. The same items were worked out for these distributions as for those in the previous paper. Also, a general idea of the natural sequence of the coefficients in the best linear estimate of the mean as the shape of the distribution undergoes change will be considered.

The mathematical formulae for this work will not be given as they are similar to those given in [3].

2. U-shaped population. The frequency distribution of a U-shaped population is

$$f(y) = \frac{3(y - \theta_1)^2}{2\theta_2^3}, \qquad \theta_1 - \theta_2 \leqq y \leqq \theta_1 + \theta_2 \tag{2.1}$$

where θ_1 is the mean and θ_2 is half the range. Standardizing the variable we get

$$f(x) = \tfrac{3}{2}x^2, \qquad -1 \leqq x \leqq +1. \tag{2.2}$$

The coefficients α_{1i} in the best linear estimates of the mean are given in Table I such that

$$\theta_1^* = \sum_{i=1}^{n} \alpha_{1i}\, y_{(i)}, \tag{2.3}$$

where $y_{(i)}$ is the ith ordered sample element.

Since

$$V(y) = \tfrac{3}{5}\theta_2^2, \tag{2.4}$$

we can estimate the standard deviation σ by $\sqrt{\tfrac{3}{5}}\, \theta_2^*$ and the coefficients can be adjusted to give the best linear estimate of the standard deviation σ^*. These adjusted coefficients for which

$$\sigma^* = \sum_{i=1}^{n} \alpha_{2i}\, y_{(i)} \tag{2.5}$$

are also shown in Table I.

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