CHARTS OF THE POWER OF THE F-TEST[1]

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1. Introduction. This paper presents charts of the power of the F-test designed to simplify entry and interpolation. The curves on which the quantity ϕ is constant are given for fixed level of significance α and power β. The coordinates are f_1 and f_2, the number of degrees of freedom in the numerator and denominator, respectively, of the F-statistic. Charts are presented for $\beta = 0.5, 0.7, 0.8, 0.9$ both for $\alpha = 0.01$ and $\alpha = 0.05$ (Figs. 1 to 8). In addition, nomograms are presented for $\alpha = 0.01, 0.05$ (Figs. 9 and 10) which make interpolation in β possible. The latter charts give linear approximations to the curves on which ϕ is constant.

The quantity ϕ is defined as $\sqrt{S_b^*/[(f_1 + 1)\sigma^2]}$, where S_b^* is the value of S_b^2 when the observable random variables are replaced by their expectations under the alternative hypothesis considered, and S_b^2 is the sum of squares in the numerator of the F-statistic.

With these charts the following question may be answered: *What experimental setup is required (what combination of f_1 and f_2), in order to obtain a specified power β against a given alternative?*

Tables of the power of the F-test have been given in two forms. Lehmer [2] tabled ϕ for fixed α, β, f_1, and f_2. On the other hand, Tang [4] tabled $P_{II} = 1 - \beta$ for fixed α, ϕ, f_1, and f_2. Essentially the same information as in Tang's tables was given, in graphical form, by Pearson and Hartley [3]. However, neither of these forms is always convenient for the design of experiments where a relation between f_1 and f_2 is desired for fixed α, β for a specified alternative hypothesis.

2. Construction of the charts. The present charts were constructed by interpolation, both numerical and graphical, in the existing tables. For $\beta = 0.5$ and 0.9, Tang's tables were used; while for $\beta = 0.7$ and 0.8, Lehmer's tables were found convenient.

Lehmer remarks that in her tables harmonic interpolation in both f_1 and f_2 is very efficient. For this reason reciprocal scales were used for f_1 and f_2. On this scale the curves of constant ϕ obtained from Lehmer's tables are nearly straight lines (see Figs. 2, 3, 6, and 7). This is especially striking for large f_1 and f_2.

Tang's tables give no entries for $f_1 > 8$. However, formula (13) of Lehmer may be used to compute ϕ for $f_1 = \infty$, while the case $f_2 = \infty$ is covered by the table of Fix [1]. As noted above, replacing the curves by straight lines for large

Received September 24, 1954; revised Feb. 7, 1956.

[1] This paper was prepared with the partial support of the Office of Naval Research and in whole or in part may be reproduced for any purpose of the United States Government.