ESTIMATION OF LOCATION AND SCALE PARAMETERS BY ORDER STATISTICS FROM SINGLY AND DOUBLY CENSORED SAMPLES

Part I. The Normal Distribution up to Samples of Size 10

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1. Introduction. Type II censored samples [3] are considered, whereby the total number of the sample elements is known but the observations for some of the extreme elements are missing. Singly censored samples are those in which only the smallest r_1 observations or the largest r_2 observations are missing, whereas samples having both r_1 smallest and r_2 largest observations missing are called doubly censored samples. This general case of estimation includes, as special cases, those estimates obtained from singly censored samples as well as those obtained by taking all the sample elements (i.e., r_1 or $r_2 = 0$ and $r_1 = r_2 = 0$).

The approach to the general case in censoring is of value not only for its numerical results. It enables the drawing of inferences concerning interesting and important patterns for the coefficients, variances, and the relative efficiencies of the estimates. These features could not be and were not revealed in the earlier, less general studies. These conclusions will be considered in Section (5).

In Part I, estimation of the mean and standard deviation from singly and doubly censored samples drawn from the normal distribution will be considered for samples $n \leqq 10$. A generalization of an alternative estimate previously given by Gupta is also obtained. In future work, it is planned to extend the tables up to samples of size $n \leqq 20$ and to include the two- and one-parameter single-exponential distributions.

Estimates of the parameters using the best linear systematic statistics are obtained by arranging the known sample elements in ascending order (i.e., $y_{(1)} \leqq y_{(2)} \leqq \cdots \leqq y_{(n)}$) and applying the method of least squares to get the best linear combination of them. The coefficients provided for these linear estimates of the ordered observations make them unbiased with minimum variance. The method used in calculation is identical with that given by Gupta in [3] with slight modifications.

2. The normal distribution. To advance the study of order statistics for the normal distribution, Hastings *et al.* [4] calculated the means, variances, and covariances of the order statistics up to samples of size 10. Godwin [1] calculated these quantities more accurately as well as extending them to more decimal places. From his tables, he was able to calculate the best linear systematic statistic of the standard deviation [2] using all the sample elements for samples of

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