which is therefore a confidence statement with a confidence coefficient greater than or equal to the confidence coefficient of (2.9). Thus, if (2.3) has a probability $1 - \alpha$, (2.9) has a probability $1 - \beta \geqq 1 - \alpha$, and if (2.9) has a probability $1 - \beta$, then (2.11) has a probability $1 - \gamma \geqq 1 - \beta$. The bounds in (2.11) are the ones obtained in [2] in a different way.

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A NOTE ON THE NORMAL DISTRIBUTION

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1. It is well known that a necessary and sufficient condition for the independence of the sample mean and variance is that the parent population be normal. This was first shown by R. C. Geary [2], and later Lukacs [3] gave a somewhat simpler proof using characteristics functions.

By using the method of Lukacs one can derive a similar theorem concerning the sample mean and the mean square successive difference.

2. Let $x_1, \cdots, x_n$ be independent and identically distributed with density $f(x)$ and mean μ and variance σ^2.

Let

$$\bar{x} = n^{-1} \sum_{j=1}^{n} x_j,$$

$$\delta_k^2 = 2^{-1}(n - k)^{-1} \sum_{j=1}^{n-k} (x_{j+k} - x_j)^2 \qquad k = 1, 2, \cdots, n - 1.$$

The following theorem can be proved:

Theorem: *A necessary and sufficient condition that $f(x)$ be the normal density is that δ_k^2 and $\bar{x}$ be independent.*

Proof: If δ_k^2 and $\bar{x}$ are independent, then we follow Lukacs [3] step for step, replacing

$$s^2 = n^{-2}[(n - 1) \sum x_\alpha^2 - 2 \sum \sum x_\alpha x_{\beta+1}]$$

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