and ρ_1 is always zero. Also,

$$V(\hat{\rho}_j) = \sigma_1^2 \Big/ \left(r - \frac{1}{k} d_j\right), \qquad j = 2, 3, \cdots, t,$$

where σ_1^2 is the within-block variance for the incomplete block design, so

$$\frac{1}{t(t-1)} E\left[\sum_{\substack{i,j \\ i \neq j}} (\hat{\tau}_i - \hat{\tau}_j)^2 \,/\, \text{all } \tau_j = 0\right] = \frac{2\sigma_1^2}{t-1} \sum_{j=2}^{t} \frac{1}{\left(r - \frac{d_j}{k}\right)},$$

which is the mean variance of a treatment difference.

For the complete block design, the mean variance of a treatment difference is $2\sigma_2^2/r$, where σ_2^2 is the variance within blocks for the complete block design.

Hence, we have the final result. The efficiency factor (EF) of an incomplete block design is equal to r times the harmonic mean of the latent roots of the matrix of coefficients of the reduced normal equations for the intrablock estimates, excluding the always-present zero root, whose characteristic vector consists of the same number repeated t times.

It may be of interest to record the view point that while the efficiency factor is a reasonable criterion of the loss due to confounding by blocking, from some points of view the generalized variance would be better. This, of course, corresponds in a certain sense to the geometric mean of the latent roots.

4. Notes on the Result. The result is interesting to the author and appears to be worth recording in the literature. It was obtained in a search for a proof of a theorem that the design with the highest efficiency factor is a balanced incomplete block design if such a design exists. To the author's knowledge, this theorem is yet to be proved.

REFERENCE

[1] Oscar Kempthorne, *The Design and Analysis of Experiments*, John Wiley and Sons, New York, 1952.

THE NULL DISTRIBUTION OF THE DIFFERENCE BETWEEN THE TWO LARGEST SAMPLE VALUES[1]

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1. Introduction. A decision procedure to select the population with the largest mean, proposed by Bose and St-Pierre [1], involves the auxiliary statistic $u = x_{(0)} - x_{(1)}$, where $x_{(0)}$ and $x_{(1)}$ are respectively the largest and second largest

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