ABSTRACTS OF PAPERS

(Additional abstracts of papers presented at the Detroit meeting of the Institute, September 7–10, 1956)

1. Further Applications of Information Theory to Multivariate Analysis and Statistical Inference, (Preliminary Report), Morton Kupperman, The George Washington University, (By Title).

A generalized statistic based on the Kullback-Leibler measure of information is defined as

$$2nI^* = 2n \int f(x, \theta^*) \log \frac{f(x, \theta^*)}{f(x, \theta_0)} \, d\lambda(x),$$

where the vector θ^* of h components is any consistent, asymptotically normal, efficient estimator and θ_0 is specified. $2nI^*$ is used to test the hypothesis H_0: The sample is from a specified multivariate multiparameter population (not necessarily normal). The asymptotic distribution of $2nI^*$ under H_0 is chi-square with h d.f. I^* is modified to test the hypothesis H_0': r ($\geqq 2$) samples are from the same general multivariate population, parameters not specified; its asymptotic distribution under H_0' is chi-square with $(r - 1)h$ d.f. Corresponding results are obtained for divergence-statistics based on the divergence $J(1, 2)$. Large-sample distributions of I^* and J^* under alternative hypotheses are approximated by noncentral chi-square distributions.

For any multivariate multiparameter distribution admitting sufficient statistics, $-\log \lambda \equiv \hat{I}$, where λ is the likelihood-ratio criterion and $\hat{I}$ uses maximum-likelihood estimators.

Information theory is applied to hypothesis testing, Pearson's chi-square test of goodness of fit, and the derivation of exact sampling distributions of sufficient statistics. It is shown that the set of sufficient estimators of population parameters appearing explicitly in any Koopman-Pitman distribution (admitting sufficient statistics) are distributed jointly in a Koopman-Pitman distribution. (Received July 19, 1956.)

2. Generalization of Thompson's Distribution, André G. Laurent, Michigan State University.

Generalization of Thompson's Distribution. 1.1.) Let $X = (X_1, \cdots, X_N)$ be $N(m, \sigma)$ distributed, $\bar{X} = \Sigma X_i/N$, $s^2 = \Sigma(X_i - \bar{X})^2/N$ and $t = (X_i - \bar{X})/s$. It is well known (W. R. Thompson. 1935) that $t^2/(N - 1)$ is Incomplete Beta distributed and that this distribution is also the conditional distribution of any X_i, given $\bar{X}$ and s. Three generalizations of that result are presented. 1.2.) If $\xi = (\xi_1, \cdots, \xi_k)'$ is a subsample from X, the p.d.f. of $t = (\xi - \bar{X})/s$ is $[1 - t(\vartheta_{ij}/N + 1/N(N - k))t']^{(N-k-3)/2}\, \Gamma[(N - 1)/2]/\pi^{k/2}\Gamma[(N - k - 1)/2]\, N^{(k-1)/2}(N - k)^{1/2}$. This provides also the conditional distribution of ξ, given $\bar{X}$ and s. 2.1.) If a vector $X = (X', \cdots, X^p)$ is $N(m, \Sigma)$ distributed and if $\xi = (\xi', \cdots, \xi^p)$ is any observation from a sample $(X) = (X_1, \cdots, X_N)$ 'with mean m' and covariance matrix S, the cond. p.d.f. of ξ, given m' and S, is $[1 - (\xi - m')S^{-1}(\xi - m')']^{(N-p-3)/2} \mid S \mid^{-1/2}[(N - 1)\pi]^{-1/2}\, \Gamma[(N - 1)/2]/\Gamma[(N - 1 - p)/2]$ 2.2.) The latter result is generalized to the case where a subsample $(\xi) = (\xi_1, \cdots, \xi_k)'$ is drawn from (X). The conditional distribution of (ξ), given m' and S, is a generalized multivariate Incomplete Beta distribution. These results make it possible to obtain and study the *U.M.V.* unbiased estimates of functions of the populations parameters, with obvious applications in the fields of S.Q.C., bombing problems, etc., and tolerance regions investigations. (Received July 23, 1956.)