TABLES FOR COMPUTING BIVARIATE NORMAL PROBABILITIES

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1. Introduction. Various tables have been published for obtaining probabilities over rectangles for correlated bivariate normal variables. Some of these tables give the probabilities as functions of three parameters (see [1], [2], and [3]). Others tabulate related two-parameter families from which these probabilities may be computed (see [3], [4], [5], [6], and [7]). The tables given here are of the latter type. They have been computed for use with a special two-dimensional interpolation scheme, which is described in Section 4. These new tabulations reduce considerably the amount of interpolation work required over that needed with previous tables. The function tabulated also eliminates an arctangent function from the formula for the bivariate normal over a region outside of a rectangle as compared with the formula for Nicholson's tabulation in [5]. Section 3 contains a derivation of the formulas given in Section 2 for using a two-parameter table to compute probabilities over rectangles. The tables given below should prove very useful, since examples where bivariate normal integrals over polygons are needed to solve practical problems abound in the literature. For example, see [6], [8], [9], and [10].

The usefulness of the $T(h, a)$ function tabulated below was also recognized by Professor Harry A. Bender, University of Rhode Island, who submitted, after this paper was received by the editor, a somewhat shorter tabulation than given here. An abstract of Professor Bender's paper appears in [15].

For h and $a > 0$, $T(h, a)$, the function tabulated, gives the volume of an uncorrelated bivariate normal distribution with zero means and unit variances over the area between $y = ax$ and $y = 0$ and to the right of $x = h$, i.e., the area shaded in Fig. 1.

Cadwell in [11] gives a method for obtaining the volume of a bivariate normal over any polygon. In Fig. 2, if AB is a side of any polygon, then the volume over the shaded area for an uncorrelated bivariate normal with zero means and unit variances is given by

$$T(h, a_2) - T(h, a_1)$$

for $a_2 > a_1$, where h is the length of the perpendicular from the origin to the line through AB and a_1h is the distance from the foot of the perpendicular, C, to B and a_2h is CA. If C lies between A and B, then the T-functions are added instead of subtracted. By composition of volumes like this, it is possible to obtain the volume over the area outside of any polygon. Section 2 includes some useful formulas for doing this.

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