ON SEQUENTIAL DESIGNS FOR MAXIMIZING THE SUM OF n OBSERVATIONS[1]

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1. Introduction. An important simple type of sequential design problem is as follows: We have two binomial random variables, X and Y, having parameters under the two hypotheses, H_1 and H_2, given by

		X	Y
(ζ)	H_1	p	q
$(1-\zeta)$	H_2	q	p,

where ζ is the a priori probability that H_1 is true. We wish to maximize the sum of n observations. The procedure for selecting an X or Y observation at each stage, of course, takes account of all the previous history.

A more realistic version of the design problem deals with the situation such that X and Y have parameters p and q, respectively, where an a priori distribution $F(p, q)$ is known. The problem holds interest for several reasons. It would appear to be one of the simplest problems in the sequential design of an experiment that can be posed; hence its analysis is a step towards obtaining a body of information relative to *specific* sequential design problems. It has not only this general interest but also, as it stands, it has applications in particular problems such as learning theory, biology, and medicine; see [1], for instance, in which applications in the latter two fields may be found. A discussion of problems of this general variety and of certain strategies has been published by Robbins [2]. More immediately, in the final section of this paper it is shown that the solution to the problem in which p has a priori distribution F and q is assumed known, explicitly obtained in Section 4, yields directly the solution of a problem in industrial inspection.

The type of problem known as the "Two-armed Bandit" is a special case of the preceding. In its "classical" formulation (whence the name), we have a slot machine with two arms, an X-arm and a Y-arm. When either arm is pulled, the machine pays off either one unit or nothing; and the probability of winning with one arm is p, and, with the other, q. A priori it is unknown which is which, but the probability ζ that it is the X-arm which has probability p of success is assumed known. One is allowed n plays, and a sequential design, or strategy, is desired which will maximize the expected winnings.

We shall use here for intuitive concreteness the gambling interpretation and terminology.

It has been conjectured for this problem that the optimal strategy is S_1 : on

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