

SOME ASPECTS OF THE ANALYSIS OF FACTORIAL EXPERIMENTS IN A COMPLETELY RANDOMIZED DESIGN¹

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1. Introduction. This paper is concerned with some aspects of the statistical analysis of factorial experiments carried out according to a completely randomized design, and is one of the joint portions of an investigation into the role and meaning of linear statistical models in the analysis of randomized experiments.

There are essentially two ways of obtaining the analysis of data obtained in a comparative experiment. One way, which is given in standard texts, is to write down a model of the type

$$y_{ijk\dots} = \mu + a_i + b_j + \dots \text{ etc.},$$

where $y_{ijk\dots}$ is the observation and the terms on the right-hand side are fixed unknown constants or random variables with specified properties. The above equation with a complete statement of all the properties of the quantities contained in it is usually called *the* model for the experiment. The texts and the literature are to the best of our knowledge, with a few exceptions to be mentioned later, bare with regard to how one determines the model, how one answers a question such as "Why not a multiplicative model?" or "Why are the a 's fixed and the b 's random?" The other way is that practiced intuitively by many experimental statisticians and described most aptly by Fisher ([3], [4], [5], [6]) in which (a) one envisages an analysis of variance of the observations from the point of view of topography, apart from treatment, such as for instance in a field experiment by rows, columns, plots within row-column cells, etc.; (b) one envisages an analysis of variance by treatments; (c) one notes how the treatments have been assigned to the experimental material, such as, for instance, factor α to rows; and (d) one therefore sees with which part of the topographical analysis any particular component of the treatment breakdown should be associated.

The second procedure cannot be regarded as fully specified by what is said above. The first procedure can only be regarded as arbitrary unless some logical basis can be given for it. It is to the problem implied in the last sentence which we have addressed our work.

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