We shall now render the introductory conditions (A)-(B) exact. First, we write

$$\text{(A)} \qquad \sigma(\zeta) \leqq \epsilon \cdot \sigma(x_i), \qquad i = 1, \cdots, h,$$

where $\epsilon \geqq 0$. Then if ϵ is small, the disturbance ζ is small in the sense that its standard deviation is small relative to the standard deviations of the explanatory variables. To give condition (B) a convenient form we observe that for given $r_1, \cdots, r_h$ there is a point $(r_1^*, \cdots, r_h^*)$ on the boundary of the ellipsoid (4) and a proportionality factor ϵ' with $0 \leqq \epsilon' \leqq 1$ such that

$$\text{(B)} \qquad r_i = \epsilon' \cdot r_i^*, \qquad i = 1, \cdots, h.$$

Then if ϵ' is small, the correlations r_i are small in the sense that the point $(r_1, \cdots, r_h)$ lies near the centre of the ellipsoid.

Thus prepared, we obtain the following

COROLLARY. *On conditions* (a) *and* (b) *of the lemma, we have*

$$| b_i - \beta_i | \leqq \epsilon \cdot \epsilon' / \sqrt{1 - R_i^2}, \qquad i = 1, \cdots, h,$$

where ϵ *and* ϵ' *are defined by* (A)-(B).

Hence if ϵ and ϵ' are of small order the specification error of the regression coefficients b_i will at most be of order $\epsilon\epsilon'$.

In the special case of one explanatory variable, $h = 1$, we have $R_1 = 0$ and $| b_1 - \beta_1 | \leqq \epsilon\epsilon'$. For example, if $\sigma(\zeta) = \frac{1}{5}\sigma(x_1)$ and $r_1 = \rho(x_1, \zeta) = \frac{1}{5}$, the specification error of b_1 cannot exceed 0, 04.

REFERENCES

1. H. WOLD IN ASSOCIATION WITH L. JURÉEN, *Demand Analysis: A Study in Econometrics*, Geber, Stockholm, 1952; and John Wiley and Sons, New York, 1953.
2. H. WOLD, "A theorem on regression coefficients obtained from successively extended sets of variables," *Skand. Aktuarietids.*, Vol. 28 (1945), pp. 181–200.

SETS OF MEASURES NOT ADMITTING NECESSITY AND SUFFICIENT STATISTICS OR SUBFIELDS[1, 2]

BY T. S. PITCHER[3]

Let X be the interval from 0 to 1 and F the field of Borel sets on X. For every $x \leqq \frac{1}{2}$, let m_x be the probability measure assigning probability $\frac{1}{2}$ to the point x and probability $\frac{1}{2}$ to the point $(x + \frac{1}{2})$ and let F_x be the subfield of F consisting of all Borel sets which contain both x and $(x + \frac{1}{2})$ or else neither. Then if M is a set of probability measures consisting of all m_x, $0 \leqq x < \frac{1}{2}$ and some measures assigning probability 0 to every point, the only set of m-measure zero

Received May 4, 1956; revised June 12, 1956.

[1] The research in this document was supported jointly by the Army, Navy, and Air Force, under contract with the Massachusetts Institute of Technology.

[2] For definitions of these concepts, see references [1] and [2].

[3] Staff Member, M.I.T., Lincoln Laboratory.