the last equality following from (9). Since $f(x)$ is an arbitrary Ω_ϕ-integrable function, this last equation, being true for all $B \varepsilon \mathfrak{F}_{s_1}$, shows that s_1 is a sufficient statistic for $\{\mu_\theta ; \theta \varepsilon \Omega_\phi\}$. But we are given that t is minimal sufficient for $\{\mu_\theta ; \theta \varepsilon \Omega_\phi\}$. Hence there is a mapping h of S_1 onto T such that $t(x) = h(s_1(x))$, $[\{\mu_\theta t^{-1}\}_\phi]$. If we now restrict x to X_ϕ it is evident that $t(x) = h(s(x))$, $[\{\mu_\theta^\phi t^{-1}\}]$, as was to be proved.

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A CENTRAL LIMIT THEOREM FOR MULTILINEAR STOCHASTIC PROCESSES

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1. Introduction. Let the random sequence $X(t)$ be observed for $t = 1, 2, \cdots$, and let $S(n) = X(1) + \cdots + X(n)$ be its consecutive sums. The random sequence may be said to obey the *classical* central limit theorem if, for any real number a,

$$\lim_{n\to\infty} \text{Prob}\left\{\frac{S(n) - ES(n)}{\sigma[S(n)]} < a\right\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-1/2x^2}\,dx. \tag{1.1}$$

Because of the importance of the central limit theorem in establishing the properties of statistical tests and estimates, it would appear that in order to develop a satisfactory theory of statistical inference for stochastic processes which are random sequences of dependent random variables, it is necessary to establish a central limit theorem for such processes. Diananda [2] has proved a central limit theorem for discrete parameter stochastic processes which are *linear* processes. We here introduce a class of stochastic processes which we call *multilinear* processes, for which we prove a central limit theorem. The results are capable of extension to the continuous parameter case, but we do not do so here.

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