ON CERTAIN TWO-SAMPLE NONPARAMETRIC TESTS FOR VARIANCES[1]

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Introduction. Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be two samples of independent observations drawn from two populations with cumulative distribution functions $F(x)$ and $G(x)$, respectively. We will assume in what follows that F and G are absolutely continuous and that they are the same in all respects except that they differ in the scale parameter. The problem considered here is that of testing the hypothesis

$$H: F = G,$$

$$A: F \neq G.$$

If the X's and the Y's come from normal populations, the usual test of significance for testing the hypothesis H is the variance ratio F-test, which is the most commonly used statistical test for comparing variances. Usually however, since little is known about the populations from which the samples are drawn, this test is used as if the assumption of normality could be ignored. It appears, however, that such is not the case. This was first pointed out by E. S. Pearson [1], who conducted certain experimental investigations. His findings were later confirmed by several other authors, especially by Geary [2] and Gayen [3]. They showed that the F-test is particularly sensitive to changes in kurtosis from the normal theory value of zero. Now, it is easy to see that the F statistic, when suitably normalised, is asymptotically distribution free. More recently, Box and Andersen [4] and [5] have studied this problem in great detail and have shown on the basis of extensive sampling experiments that the F statistic so normalised is insensitive to departures from normality, at least for large samples. Very recently attempts have also been made to construct nonparametric tests, particularly by Mood [6] and Lehmann [7].

The test proposed by Mood is similar to the variance ratio F-test with ranks replacing the original observations. He has also computed the asymptotic relative efficiency of the test with respect to the F-test for normal alternatives. In this paper, we will derive a general formula for the asymptotic relative efficiency of Mood's test with respect to the F-test for scalar alternatives but almost arbitrary continuous distributions.

The test proposed by Lehmann is essentially of the Wilcoxon-Mann-Whitney type (see [8] and [9]) applied to all possible differences between the X's and

Received January 29, 1956.

[1] This research was performed while the author was at the Statistical Laboratory, University of California, Berkeley, and was supported by the Office of Ordnance Research, United States Army, under Contract DA-04-200-ORD-171.