ON THE ESTIMATION OF AUTOCORRELATION IN TIME SERIES

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1. Introduction. In a recent paper, F. H. C. Marriott and J. A. Pope [8] investigated, in some special cases, the bias arising in the estimation of the autocorrelation function of a discrete-parameter stochastic process when its mean is not known. M. G. Kendall [4] developed a general method for the determination of this bias in the case of an arbitrary Gaussian process.

The removal of the mean from a stochastic process may be regarded as a particular case of the elimination of a polynomial trend. The object of the present paper is to determine how the removal of this trend affects both the biases and the covariances of the estimators of the covariances and of the autocorrelation coefficients; it is not assumed that the process is necessarily Gaussian.

In the two papers mentioned above, the passage from the estimation of the covariances to that of the correlation coefficients was achieved by what may be called the method of statistical differentials. The estimator $\hat{\rho}_{k,N}$ of ρ_k was regarded as a function of certain covariance estimators and, in the derivation of relevant formulae, the difference $\hat{\rho}_{k,N} - \rho_k$ was replaced by the first differential of this function. The general validity of this kind of argument needs to be clarified, as remarked by Kendall himself in the last paragraph of his note. The same applies to the derivation of cov $(\hat{\rho}_{k,N}, \hat{\rho}_{l,N})$ by Bartlett [1] in the case in which there is no trend. In order to make rigorous this kind of argument, we prove a general theorem conceived in the same spirit as a proposition given by Cramér ([3], pp. 353–356) for functions of sample moments, and justifying the use of the method of "statistical differentials" under specified assumptions.

2. Basic definitions and assumptions.

Assumption 1. In what follows it will be assumed that $\{y_t\}$ is a discrete-parameter stochastic process composed of a determinate polynomial trend $f_m(t)$ of degree at most m, and of a linear stochastic process $\{x_t\}$:

$$y_t = f_m(t) + x_t \qquad (t = 0, \pm 1, \pm 2, \cdots), \tag{2.1}$$

where $\{x_t\}$ is of the form of

$$x_t = \sum_{s=0}^{\infty} h_s \epsilon_{t-s}, \tag{2.2}$$

the series $\sum_{t=0}^{\infty} h_t$ being absolutely convergent, and $\{\epsilon_t\}$ being a wide-sense stationary process with zero means:

$$E(\epsilon_t) = 0, \qquad E(\epsilon_t^2) = \sigma^2, \qquad E(\epsilon_t \epsilon_s) = 0 \qquad \text{for } t \neq s. \tag{2.3}$$

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