# MULTISTAGE STATISTICAL DECISION PROCEDURES[1]

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**1. Introduction.** A class of problems which arise in a variety of forms can be formulated as follows: We are requested to make periodic decisions of the same type but based on an increasing amount of information. Suppose we have a collection $D_k$ of decision procedures for the $k$th stage, given the amount of information available to us at that stage, and suppose that the procedures of $D_k$ are admissible under the assumption that the $k$th-stage decision is all that is required of us. Is it then true that when we have to prescribe decision procedures $d_1, d_2, \cdots, d_n$ for each stage, we obtain an admissible class by taking an arbitrary procedure from each $D_k$? The answer turns out to be no in a large class of such decision problems which we consider here. This means that by planning our whole sequence of decision procedures in advance we are able to do better on an average than if we were to make each decision as it arises. The present paper is devoted to the problem of prescribing rules which tell how the single-stage decision procedures $d_k$ should interlock with one another so as to give a minimal complete class of decision procedures for the multistage statistical decision problem. This problem is similar to the classical sequential decision problems which do not fix in advance the number of stages. In many respects the problem formulated here is simpler than the classical sequential decision problem, and sharper results are obtained—e.g., minimal complete classes of statistical decision procedures are determined.

In order to illustrate the nature of this type of decision problem and its analysis, we might look at the following simple example: A biased coin is tossed, and the player is required to call heads or tails, being paid one unit for a correct call and nothing for an incorrect one. The first call of the player is made in complete ignorance, but for the $n$th play he has the evidence of the first $(n - 1)$ tosses on which to make his call. To prescribe the classes $D_n$ of strategies admissible for a single stage is to consider the problem of making the $n$th call on the basis of the first $(n - 1)$ outcomes, the first $(n - 1)$ calls having been forgotten. We then have the game in which nature chooses the bias $p$ on the coin, the player observes a random variable $z$ binomially distributed with parameter $p$ and must choose between two actions with loss function $-p$ and $(p - 1)$. Here a decision procedure or strategy for the player consists of a function $\phi(z)$ which gives the probability[2] of calling heads if $z$ is the number of heads that have previously been observed. Problems of this sort have been considered in [1]

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[2] We are thus allowing the player its use of a randomized strategy.