A MOVING SINGLE SERVER PROBLEM

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1. Introduction. An assembly line moving with uniform speed has items for service spaced along it. The single server available moves with the line while serving and against it with infinite velocity while transferring service to the next item in line. The line has a barrier in which the server may be said to be "absorbed" in the sense that service is disabled if the server moves into the barrier. The problem solved here is the following: given that a server with exponentially distributed service time starts service on the first item when it is T time units away from the barrier, what is the probability $p(k, T)$ that it completes k items of service before absorption? This is the same as determining the generating function

$$P(x, T) = \sum_{k=0}^{\infty} p(k, T)x^k. \tag{1}$$

The referee has pointed out to us an identification of this problem with that of finding the number of units of service in a busy period for the usual (stationary) single server. This may be seen as follows.

Take $\tau(t)$ as the distance from the barrier at time t, so that $\tau(0) = T$. Take the spacing between items as an independent random variable with distribution function $B(t)$. Then the graph of $\tau(t)$ as in Fig. 1 consists of lines of unit slope interrupted by jumps having the distribution $B(t)$ and occurring at t-epochs determined by the exponential distribution of service time. The graph ends when $\tau(t) = 0$ for the first time, when service is disabled.

Now consider the queueing system with a single server, Poisson arrivals, and distribution of service times $B(t)$. Take $\tau(t)$ as the waiting time of a *virtual* arrival at time t. Then the graph of $\tau(t)$ for a single busy period of the server is exactly as in Fig. 1 if the first customer served has a service time which is *given* to be T.

Note that one problem is turned into the other by interchanging service and arrival variables.

Busy periods were first considered by E. Borel [2] for the case of constant service time and with main interest in the number served, exactly as here, but with the first customer's service time the same constant as all others. Turning to the length of the busy period, D. G. Kendall [4] generalized Borel's result to arbitrary service time distribution by transforming it into a question concerning a branching process. Kendall's functional equation was carefully derived by L. Takacs [7], who also obtained a similar equation for the generating function for the number served in a busy period (with no condition on the first customer) for

Received July 5, 1956; revised October 5, 1956.