$X_{N-i+1} - X_i$ by $w_{(i)}$ and $w_{(1)} = w$. The unbiased estimate of the type $s' = k' (\sum w_{(i)})$, where the summation is over the subset of all $w_{(i)}$ which gives minimum variance, is indicated in Table II. The column headed "Eff." refers to the comparison with the unbiased sample standard deviation. The final column gives the ratio of the variance of the best linear systematic statistic as given in [2] to the variance of s'. By examining this ratio we can see that the loss in efficiency due to the use of "zero or one" weights for each range rather than the optimum weights given in [2], is not great.

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THE INDIVIDUAL ERGODIC THEOREM OF INFORMATION THEORY[1]

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1. Introduction. Information theory is largely concerned with stationary stochastic processes $\cdots x_{-1}, x_0, x_1, \cdots$ taking values in a finite "alphabet," $a_1, \cdots, a_s$. In addition, it is usually assumed that the processes are ergodic, that is to say, the shift operator T, defined on the sequence space Ω of the process by shifting each coordinate of a sequence once to the right, is metrically transitive with respect to the probability measure p on Ω.

A question of importance in information theory regarding these processes is the nature and existence, in some sense, of the expression

$$\text{(a)} \qquad \lim_n \left(-\frac{1}{n} \log_2 p(x_0, \cdots, x_{n-1}) \right).$$

In 1948 Shannon [1] showed that for stationary, ergodic Markov chains (a) exists as a limit in probability and is equal to a constant. This limiting constant was termed by Shannon the "entropy" of the process. In 1953 McMillan [2] lifted the restriction to Markov chains and proved that if the process is merely stationary and ergodic, then (a) exists as a limit in L_1 mean and is constant. The purpose of this note is to prove that under the same conditions the limit (a) exists almost surely (a.s.).

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