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NON-PARAMETRIC UP-AND-DOWN EXPERIMENTATION[1]

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1. Introduction. Let $Y(x)$ be a random variable such that $P(Y(x) = 1) = F(x)$ and $P(Y(x) = 0) = 1 - F(x)$ where $F(x)$ is a distribution function. It is sometimes of interest, as in sensitivity experiments, to estimate a given quantile of $F(x)$ with observations distributed like $Y(x)$ where the choice of x is under control. A procedure for estimating the median was suggested by Dixon and Mood [2]. The validity of their procedure depends on the assumption that $F(x)$ is normal. Robbins and Monro [6] suggested a general scheme which can be used for estimating any quantile and which imposes no parametric assumptions on $F(x)$. Their method does assume, however, that the range of possible experimental values of x is the real line. In practice, this will not be the case. Limitations on the precision of measuring instruments, or natural limitations such as when x is obtained by a counting procedure, will usually restrict the experimental range of x to a set of numbers of the form

$$a + hn(-\infty < a < \infty, h > 0, n = 0, \pm 1, \cdots).$$

In this note we suggest a non-parametric procedure for estimating any quantile of $F(x)$ on the basis of quantal response data when, experimentally, x is restricted to the form $a + hn$.

For convenience we assume $a = 0$, $h = 1$. Suppose we wish to estimate that value of $x = \theta$ such that $F(\theta - 0) \leqq \alpha \leqq F(\theta)$, $\frac{1}{2} \leqq \alpha < 1$. If $0 < \alpha \leqq \frac{1}{2}$ or $a \neq 0$ or $h \neq 1$ the necessary modifications will be apparent. The experimental procedure is as follows: choose x_1 arbitrarily. Recursively, let

$$\begin{aligned} x_n &= x_{n-1} - 1, \qquad \text{with probability } \frac{1}{2\alpha} \text{ if } y_{n-1} = 1, \\ &= x_{n-1} + 1, \qquad \text{with probability } 1 - \frac{1}{2\alpha} \text{ if } y_{n-1} = 1, \\ &= x_{n-1} + 1, \qquad \text{with probability 1 if } y_{n-1} = 0. \end{aligned} \tag{1}$$

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