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PERCOLATION PROCESSES: LOWER BOUNDS FOR THE CRITICAL PROBABILITY

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1. Introduction. A percolation process is the spread of a fluid through a medium under the influence of a random mechanism associated with the medium. This contrasts with a diffusion process, where the random mechanism is associated with the fluid. Broadbent and Hammersley [1] gave examples illustrating the distinction.

Here we shall consider a *medium* consisting of an infinite set of *atoms* and *bonds*. A bond is a path between two atoms: it may be *undirected* (in which case it will allow passage from either atom to the other) or it may be *directed* (in which case it will allow passage from one atom to the other but not vice versa). Two atoms may be linked by several bonds, some directed and some undirected. Broadbent and Hammersley [1] dealt with *crystals*, i.e., media in which the atoms and bonds satisfied three postulates denoted by $P1$, $P2$, and $P3$. Here, however we shall dispense with $P1$ and a part of $P3$; and our surviving assumptions are:

$P2$. The number of bonds *from* (but not necessarily *to*) any atom is finite.

$P3$(a). Any finite subset of atoms contains an atom *from* which a bond leads to some atom not in the subset.

With this medium we associate the following *random mechanism*: each bond has an independent probability p of being *undammed* and $q = 1 - p$ of being *dammed*. *Fluid*, supplied to the medium at a set of *source atoms*, spreads along undammed bonds only (and in the permitted direction only for undammed directed bonds) and thereby *wets* the atoms it reaches. Associated with each atom A, there is a *critical probability* $p_d(A)$, defined as the supremum of all values of p such that, when A is the only source atom, A wets only finitely many atoms with probability one. We seek lower bounds for p_d.

An *n-stepped walk* is an ordered connected path along n bonds, each step being in a permitted direction along its bond and starting from the atom reached by the previous step. Walks (as opposed to fluid) may traverse dammed bonds: a walk is dammed or undammed according as it traverses at least one or no

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