A NOTE ON BIBDS

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It is well known that the parameters (v, b, r, k, λ) of a balanced incomplete block design (BIBD) satisfy the relations

$$bk = rv, \tag{1}$$

$$\lambda(v - 1) = r(k - 1), \tag{2}$$

$$r - \lambda = rk - \lambda v. \tag{2'}$$

Fisher [2] proved that

$$b \geqq v, \tag{3}$$

and Bose [1] showed that for a resolvable BIBD one has

$$b \geqq v + r - 1. \tag{4}$$

Nair [3] proved the inequality

$$b \geqq 1 + \frac{k(r - 1)^2}{(r - k) + \lambda(k - 1)} \tag{5}$$

for any BIBD, and

$$b \geqq \frac{rk(r - 1)}{(r - k) + \lambda(k - 1)} \tag{6}$$

for a resolvable BIBD. While it was originally claimed that (5) and (6) are sharper results than (3) and (4), it is the purpose of this note to show that this is not so; (5) and (6) are completely equivalent to (3) and (4).

We first put (4) in a neater form by writing it as $(b - r)k \geqq k(v - 1)$; using (1) and (2),

$$rv - \lambda(v - 1) - r \geqq k(v - 1),$$

$$(v - 1)(r - \lambda) \geqq k(v - 1).$$

Since $v - 1 > 0$, (4) is equivalent to

$$r - \lambda \geqq k. \tag{7}$$

We now take Nair's inequality (6); using (1), it is equivalent to $v(r - k + \lambda k - \lambda) \geqq k^2(r - 1)$. Applying (2′), and the fact that $v - k > 0$, we then obtain

$$k(r - 1)(v - k) \geqq \lambda v(v - k),$$

$$k(r - 1) \geqq \lambda v,$$

$$r - \lambda \geqq k.$$

This demonstrates the equivalence of (4), (6), and (7).

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